Math 1314
Lesson 19: Numerical Integration

For more complicated functions, we will use GeoGebra to find the definite integral. These will include functions that involve the exponential function, logarithms, rational functions and powers of binomials. You should be able to find the definite integral of any polynomial function using the rules for integration, but for everything else, you will need to use technology.

Example 1: Evaluate $\int_0^2 (6x^2 - 4e^x)dx$ $= -9.5562$

Step 1: Enter the function in the input line

Step 2: Find the definite integral. What's the command? What are the inputs?

Integral [function, start, end]

Integral [f, 0, 2]

Example 2: Evaluate: $\int_1^4 \left(\frac{1}{x} - \frac{1}{x^2}\right)dx$ $= .6363$

$\int_a^b f(x)\,dx$

$-\int_b^a f(x)\,dx$

Integral [g, 1, 4]

Example 3: Evaluate: $\int_2^5 \frac{2x^2 - 4x + 6}{x}dx$ $= 14.4977$

Integral [h, 2, 5]

Example 4: Evaluate: $\int_0^3 (e^x - x + 1)dx$ $= 17.5855$

Integral [x, 0, 3]

Example 5: Evaluate $\int_0^3 4x\left(x^2 - 3\right)^5 dx$ $= 15309$

Example 6: Evaluate $\int_1^2 \frac{x^2}{3x^3 + 6} dx$ $= 0.1338$

Midterm 2 ↑

Final ↓

We can use integration to solve a variety of problems.

$\int_a^b r(x)\,dx \rightarrow R(x)\Big|_a^b$

1 Suppose we are given the rate at which a worker can produce items along an assembly line. This is a derivative – a rate of change. When we find the antiderivative, we are finding a function that gives us the total number of items that can be produced. So in this instance, the area under the curve will give total production.

2 The marginal cost function is the derivative of the cost function. When we find the antiderivative of the marginal cost function, we have the total cost function, and the area under the curve will give total cost for the given production levels.

3 The velocity function is the derivative of the position function. When we find the antiderivative of the velocity function, we have the position function, and the area under the curve will give the total distance traveled.

So, when you "go backwards" from a function that gives a rate of change, you get a function that gives the total quantity for that function. Then the area under the curve is the total quantity for that function between two given numbers, *a* and *b*.

Example 7: A company purchases a new machine for which the rate of depreciation is given by $10{,}000(t-6)$. How much value is lost over the first three years that the machine is in use?

$\rightarrow [0, 3]$

$\int_0^3 10000(t-6)\,dt = -135000$

$\boxed{\$135000}$

Example 8: An office supply company estimates that the daily marginal cost associated with producing one of its products can be modeled by the function $C'(x) = 0.000006x^2 - 0.008x + 8$. The fixed costs per day associated with producing this product is $600, so even if no items are produced, the company incurs these expenses.

(A) Find the total cost of producing the first 500 units.
(B) Find the total cost for producing the 201st thorough 400th units.

A. $[0, 500]$ $\int_0^{500} (0.000006x^2 - 0.008x + 8)\,dx$

$= 3250 \rightarrow$ variable cost

total cost $= 600 + 3250 = \boxed{\$3850}$

B $[201, 400]$ $\int_{201}^{400} (0.000006x^2 - 0.008x + 8)\,dx = 1225.36$

$\boxed{\$1225^{36}}$

i.e. derivative

Example 9: A study of worker productivity shows that the rate at which a typical worker can produce widgets on an assembly line can be expressed as $N(t) = -3t^2 + 12t + 15$ where t gives the number of hours after a worker's shift has begun. Determine the number of widgets a worker can produce during the first hour of his/her shift. Determine the number of widgets a worker can produce during the last hour of a five hour shift.

shift [0, 5]

[0,1] [1,2] [2,3] [3,4] [4,5]

1st 4th last

First hour

$$\int_0^1 (-3t^2 + 12t + 15)\,dt = 20$$

could do by hand

Last hour

$$\int_4^5 (-3t^2 + 12t + 15)\,dt = 8$$

$$\int_2^7 (3x^2 - 0.4x - 6)\,dx$$

Integral [f, 2, 7] = answer

$f(x) = 3x^2 - 0.4x - 6$

Example 10: A city's rate of electricity consumption is expected to grow according to the function $R(t) = 40e^{0.05t}$ where consumption is measured in million kilowatt-hours per year. What should be the total production of electricity over the next three years in order to meet the projected demand?

ROC

[0, 3]

$$\int_0^3 40e^{0.05t}\,dt$$

Integration gives total # needed.

① $f(t) = 40e^{0.05t}$ Enter in GGB

② Integral [f, 0, 3] = 129.4674

③ Answer question

$\boxed{129.4674 \text{ million kWhours}}$

[0, 30]

ROC

Example 11: The population of a city is projected to grow at the rate of $18\sqrt{t+1}\ln\sqrt{t+1}$ thousand people per year t years from now. If the current population is 1,500,000, what will be the population 30 years from now?

① enter $f(t) = 18\sqrt{t+1} * \ln\sqrt{t+1}$

② Integral [f, 0, 30] = 2869.8487

③ Integral gives amt. of population growth

2869.8487 thousand = 2,869,849

$\underbrace{2{,}869{,}849}_{\text{growth}} + \underbrace{1{,}500{,}000}_{\text{current}} = \boxed{4{,}369{,}849}$

ROC

Example 12: Suppose you are driving a car and that your velocity can be approximated by $v(t) = 2t\sqrt{25-t^2}$, where t is measured in seconds and v is measured in feet per second. How far will you travel in the 5 seconds from $t = 0$ to $t = 5$?

$\boxed{\int_0^5 2t\sqrt{25-t^2}\,dt}$

① enter $v(t)$ into GGB

② Integral [v, 0, 5] = 83.3333

③ $\boxed{83.3333 \text{ ft}}$

ROC

Example 13: The marginal daily profit function associated with production and sales of a video game is estimated to be $P'(x) = -0.0003x^2 + 0.04x + 17$ where x is the number of units produced and sold daily and $P'(x)$ is measured in dollars per unit. Find the additional daily profit realizable if production and sales is increased from 200 units per day to 300 units per day.

$\int_{200}^{300} (-0.0003x^2 + 0.04x + 17)\,dx$ [200, 300]

① Enter $P'(x)$ in GGB

② Integral [P', 200, 300] = 800

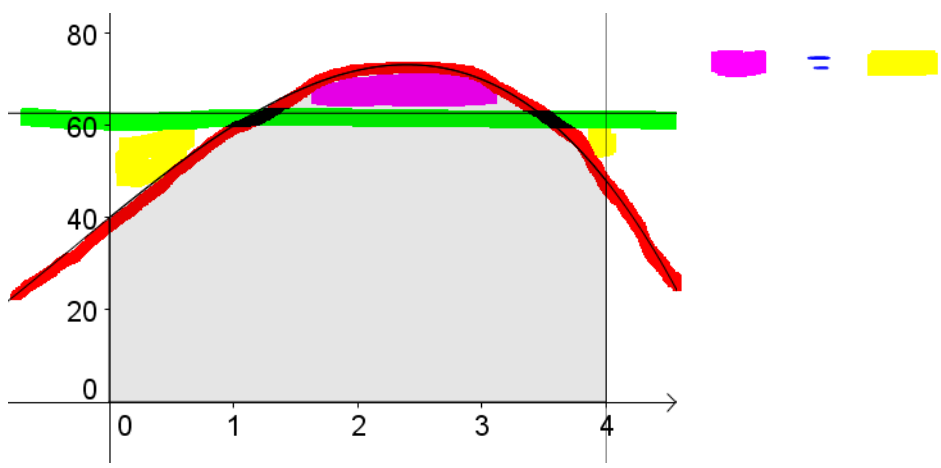
③ $\boxed{\$800}$

The Average Value of a Function

We can use the definite integral to find the average value of a function**.**

Suppose *f* is an integrable function on the interval [*a*, *b*]. Then the average value of *f* over the interval is $\frac{1}{b-a}\int_a^b f(x)dx$.

The graph shown below illustrates the average value. In this graph, the area under the curve from $x = 0$ to $x = 4$ is exactly the same as the area of the rectangle formed by the *x* and *y* axes, the line $x = 4$ and the line $y = 62.6667$. The value 62.6667 is the "average value" of the function and was computed using the formula $\frac{1}{b-a}\int_a^b f(x)dx$.



Example 14: Find the average value of $f(x) = \sqrt{x}$ over the interval [1, 16].

a b

$\frac{1}{b-a}\int_a^b f(x)\,dx$

Set it up

$\frac{1}{16-1}\int_1^{16}\sqrt{x}\,dx$

$\boxed{\frac{1}{15}\int_1^{16}\sqrt{x}\,dx}$

① enter f(x) in GGB f(x) = sqrt(x)

② Integral [f, 1, 16] = 42

③ Multiply by $\frac{1}{15}$ $\frac{1}{15}(42) = \boxed{2.8}$

Example 15: Find the average value of $f(x) = x^2 - 3x + 5$ on $[\overset{a}{2}, \overset{b}{5}]$.

Set up

$$\frac{1}{5-2}\int_2^5 (x^2 - 3x + 5)\,dx$$

$$\frac{1}{3}\int_2^5 (x^2 - 3x + 5)\,dx$$

① enter into GGB

② Integral [f, 2, 5] = 22.5

③ Multiply by $\frac{1}{3}$ $\quad \frac{1}{3}(22.5) = \boxed{7.5}$

Example 16: The sales of ABC Company in the first t years of its operation is approximated by the function $S(t) = t\sqrt{0.4t^2 + 4}$ where $S(t)$ is measured in millions of dollars. What were the company's average annual sales over its first five years of operation?

so average value $\quad [0, 5]$

Set it up $\quad \frac{1}{5-0}\int_0^5 S(t)\,dt$

$$\frac{1}{5}\int_0^5 t\sqrt{0.4t^2+4}\,dt$$

① enter S(t) into GGB

② Integral [S, 0, 5] = 36.986

③ Multiply by $\frac{1}{5}$ $\quad \frac{1}{5}(36.986) = 7.3972$

$\boxed{\$7.3972 \text{ million}}$

Example 17: The median price of a house in a city in Arizona can be approximated by the function $f(t) = t^3 - 7t^2 + 17t + 190$ where the median price is given in thousands of dollars and t is given as the number of years since 2000. This function has been shown to be valid for the years 2000 to 2005. Determine the average median price of a home in this city during this time period.

so ... average value $\quad$ interval $[0, 5]$

Set it up $\quad \frac{1}{5-0}\int_0^5 f(t)\,dt$

$$\frac{1}{5}\int_0^5 (t^3 - 7t^2 + 17t + 190)\,dt$$

① enter f(t) into GGB

② Integral [f, 0, 5] = 1027.0835

③ Multiply by $\frac{1}{5}$ $\quad \frac{1}{5}(1027.0835) = 205.4167$

$\boxed{\$205.4167 \text{ thousand}}$

$\boxed{\$205{,}417^{00}}$