Section 6.2: Introduction to Probability

The ratio $\frac{m}{n}$ is the **relative frequency** of an event E that occurs m times after n repetitions.

Note: The probability of an event is a number that lies between 0 and 1, inclusive.

If S = {s_1, s_2 ,…, s_n } is a finite sample space with n outcomes, then the events {s_1}, {s_2},…, {s_n} are called **simple events** of the experiment.

Once probabilities are assigned to each of these simple events, we obtain a probability distribution.

The probabilities, P(s_1), P(s_2),…, P(s_n) have the following properties:

1. $0 \le P(s_i) \le 1$, $i = \{1, 2, 3, \dots, n\}$
2. $P(s_1) + P(s_2) + \cdots + P(s_n) = 1$
3. $P(s_i \cup s_j) = P(s_i) + P(s_j)$, $i \neq j$ and $i, j = 1,2,3, \dots n$

Example 1: A fair die is cast. List the simple events.

S = { 1 , 2 , 3 , 4 , 5 , 6 }

P(x) = $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$

A sample space in which the outcomes of an experiment are equally likely to occur is called a uniform sample space. Let S = {s_1, s_2,…, s_n} be a uniform sample space. Then

$$P(s_1) = P(s_2) = \cdots = P(s_n) = \frac{1}{n}$$

Finding the probability of an Event E:
1. Determine the sample space S.
2. Assign probabilities to each of the simple events of S.
3. If E={ s_1, s_2,…, s_k} where {s_1}, {s_2},…, {s_k} are simple events then

$$P(E) = P(s_1) + P(s_2) + \cdots + P(s_k)$$

Note: If E = Ø then P(E) = 0.

E = Numbers bigger 4 = 5, 6

$P(E) = P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

Popper 1 is A

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Example 2: The accompanying data were obtained from a survey of Americans who were asked: How safe are American-made consumer products

Rating	Number of Respondents
Very Safe	76
Somewhat safe	244
Not too safe	60
Not safe at all	8
Don't know	12

Find Total

400 people

Find the probability distribution associated with this experiment.

	VS	SS	NTS	NSA	DK
	$\frac{76}{400}$	$\frac{244}{400}$	$\frac{60}{400}$	$\frac{8}{400}$	$\frac{12}{400}$
P(X)	0.19	0.61	0.15	0.02	0.03

Example 3: A pair of fair dice is cast. What is the probability that

a. the sum of the numbers shown is less than 5?

6 scenarios

$P(\text{Sum less 5}) = \frac{6}{36} = \boxed{0.1667}$

b. at least one 6 is cast?

11 scenarios

$P(\text{At least one 6}) = \frac{11}{36} = \boxed{0.3055}$

c. you roll doubles?

6 scenarios

$P(\text{Double}) = \frac{6}{36} = \boxed{0.1667}$

SECOND DIE

FIRST DIE	1	2	3	4	5	6	
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	6
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	6

36

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Example 4: If one card is drawn from a well-shuffled standard 52-card deck, what is the probability that the card drawn is

a. A club?

↖ 13 of them

$P(\text{club}) = \frac{13}{52} = \boxed{0.25}$

b. A red card?

↖ 13 Hearts
13 Diamonds

$P(\text{Red}) = \frac{26}{52} = \boxed{0.50}$

c. A seven?

↖ One from each suit

$P(7) = \frac{4}{52} = \boxed{0.0769}$

d. A face card?

K, Q, J

↖ 3 from each suit

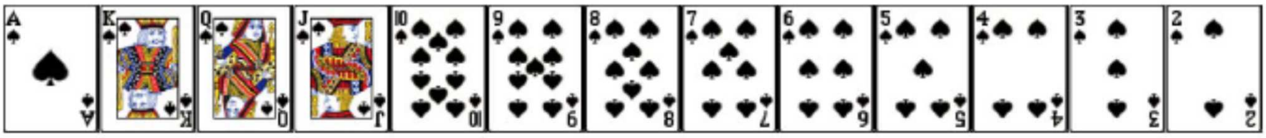
$P(\text{Face card}) = \frac{12}{52} = \boxed{0.2308}$

e. A black 9?

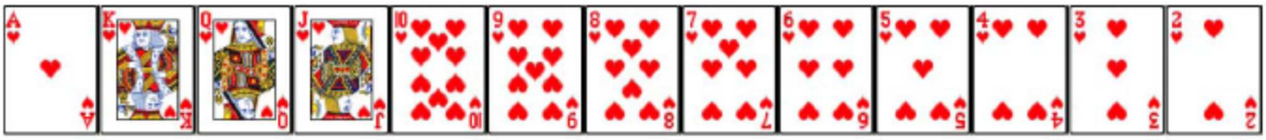
↖ 1 from Club
1 from Spade

$P(\text{Black } 9) = \frac{2}{52} = \boxed{0.0385}$

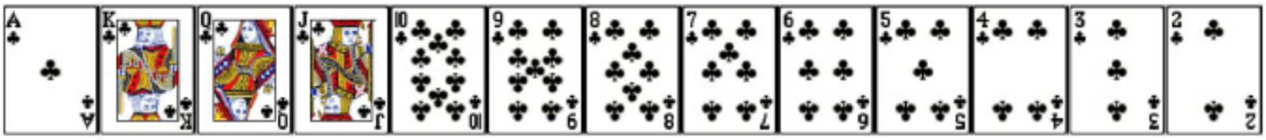
13 Spades
Black



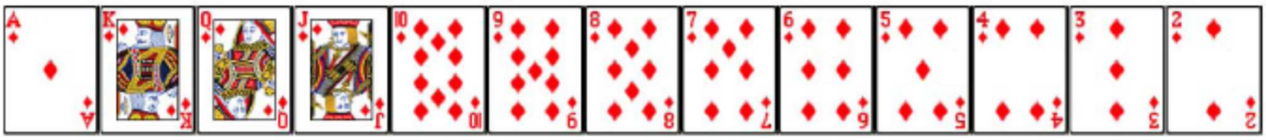
13 Hearts
Red



13 Clubs
Black



13 Diamonds
Red



K, Q, J – Face Cards

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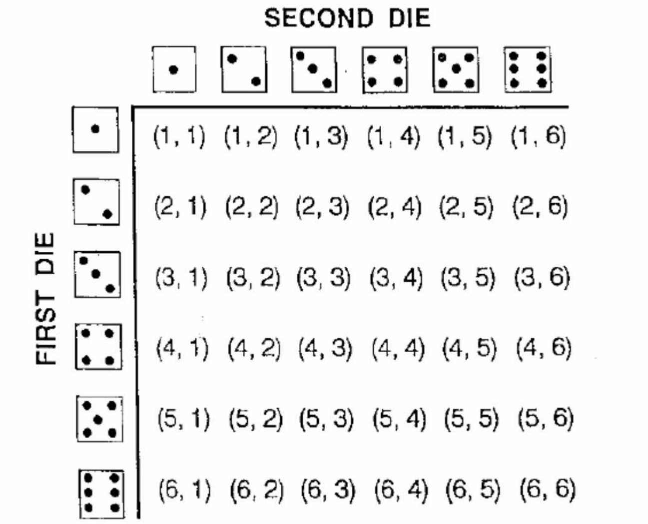
Popper 3: A pair of fair dice is cast. What is the probability that the sum of the numbers falling uppermost is 7?

a. 0.5833
b. 0.1667
c. 0.1944
d. None of the above



Example 5: A survey was taken in a certain community about the number of the radios in the house, the probability distribution was constructed:

Number of Radios	0	1	2	3
Probability	0.01	0.09	0.53	0.37

What is the probability of a house chosen at random from this community having,

a. 1 or 2 radios? $P(1) + P(2)$

$= 0.09 + 0.53 = \boxed{0.62}$

b. more than 1 radio? $= P(2) + P(3)$

$= 0.53 + 0.37 = \boxed{0.90}$

c. not even one radio? $= P(0)$

$= 0.01$

Popper 4: If one card is drawn from a well-shuffled standard 52-card deck, what is the probability that the card drawn is a red six?

a. 1/52
b. 1/26
c. 1/13
d. 2/13
e. None of the above

Example 6: Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be the sample space associated with an experiment having the following probability distribution:

Outcome	s_1	s_2	s_3	s_4	s_5
Probability	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{20}$	$\frac{2}{5}$	$\frac{1}{4}$

If $G = \{s_2, s_5\}$, $H = \{s_1, s_2, s_3\}$, and $I = \{s_1, s_4\}$. Find the probability.

a. $P(G)$ $= P(s_2) + P(s_5)$

$\frac{1}{5} + \frac{1}{4} = \frac{9}{20} = \boxed{0.45}$

b. $P(G \cup H)$ $= P(s_1) + P(s_2) + P(s_3) + P(s_5)$

$\{s_1, s_2, s_3, s_5\}$

$\frac{1}{10} + \frac{1}{5} + \frac{1}{20} + \frac{1}{4} = \frac{12}{20} = \boxed{0.60}$

c. $P(I \cap G)$ $= P(\emptyset)$

$\{\emptyset\}$

$\boxed{0}$

Popper 5: How many quizzes are due this weekend?

a. 0
b. 1
c. 2
d. 3

Popper 2 is B