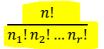
1

Math 1313 Section 5.4 Section 5.4: Permutations and Combinations Definition: n-Factorial For any natural number n, $n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$ 0! = 1A permutation is an arrangement of a specific set where the order in which the objects are arranged is important. Formula: $P(n,r) = \frac{n!}{(n-r)!}$, $r \le n$ where *n* is the number of distinct objects and *r* is the number of distinct objects taken *r* at a time. Formula: Permutations of n objects, not all distinct $i \in \mathbb{N}$ and $i \in \mathbb{N}$.

Given a set of n objects in which n_1 objects are alike and of one kind, n_2 objects are alike and of another kind,..., and, finally, n_r objects are alike and of yet another kind so that

 $n_1 + n_2 + \dots + n_r = n$

then the number of permutations of these n objects taken n at a time is given by



A **combination** is an arrangement of a specific set where the order in which the objects are arranged is not important.

Formula: $C(n, r) = \frac{n!}{r!(n-r)!}, r \le n$

where n is the number of distinct objects and r is the number of distinct objects taken r at a time.

Example 1: You are in charge of seating 5 honored guests at the head table of a conference. How many seating arrangements are possible if the 8 chairs are on one side of the head table?

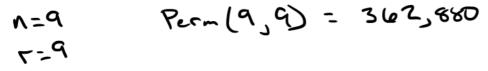
$$n=8 \qquad \text{Line } \cup_{p} \rightarrow \text{Pern.}$$

$$r=5 \qquad P(8,5) = 6720$$

$$chirs \qquad 1 \land People$$

$$8 \qquad 7 \qquad 6 \qquad 5 \qquad 4$$

Example 2: Find the number of ways 9 people can arrange themselves in a line for a group picture.



Popper 3: You are in charge of seating 4 honored guests at the head table of a conference. How many seating arrangements are possible if the 10 chairs are on one side of the head table?

- a. 5040 b. 210 ∧ ¬.♥
- c. 40
- d. None of the above

Example 3: An organization has 30 members. In how many ways can the positions of president, vicepresident, secretary, treasurer, and historian be filled if not one person can fill more than one position?

--4

Example 4: An organizations needs to make up a social committee. If the organization has 25 members, in how many ways can a 10 person committee be made?

Order is NOT important - Combination 1-25 C(25,10) = 3,268,740 C= 10

Example 5: If there are 40 contestants in a beauty pageant, in how many ways can the judges award 1st prize and 2nd prize if not one person can be awarded 1st and 2nd?

n = 40

Rank + Perm.

5=2

P(40,2) = 1560

Example 6: How many permutations can be formed from all the letters in the word MISSISSIPPI.

Popper 4: In a production of *West Side Story*, eight actors are considered for the male roles of Tony, Riff, and Bernardo. In how many ways can the director cast the male roles? **What type of problem is this?**

- a. Combination
- b. Permutation

Example 7: A museum of fine arts owns 8 paintings by a given artist. Another fine arts museum wishes to borrow 3 of these paintings for a special show. How many ways can 3 paintings be selected for shipment out of the 8 available?

Order does NOT matter $\Lambda = 8$ r=3 0 (8,3) = 50

Example 8: A certain company has to transfer 4 of its 10 junior executives to a new location, how many ways can the 4 executives be chosen?

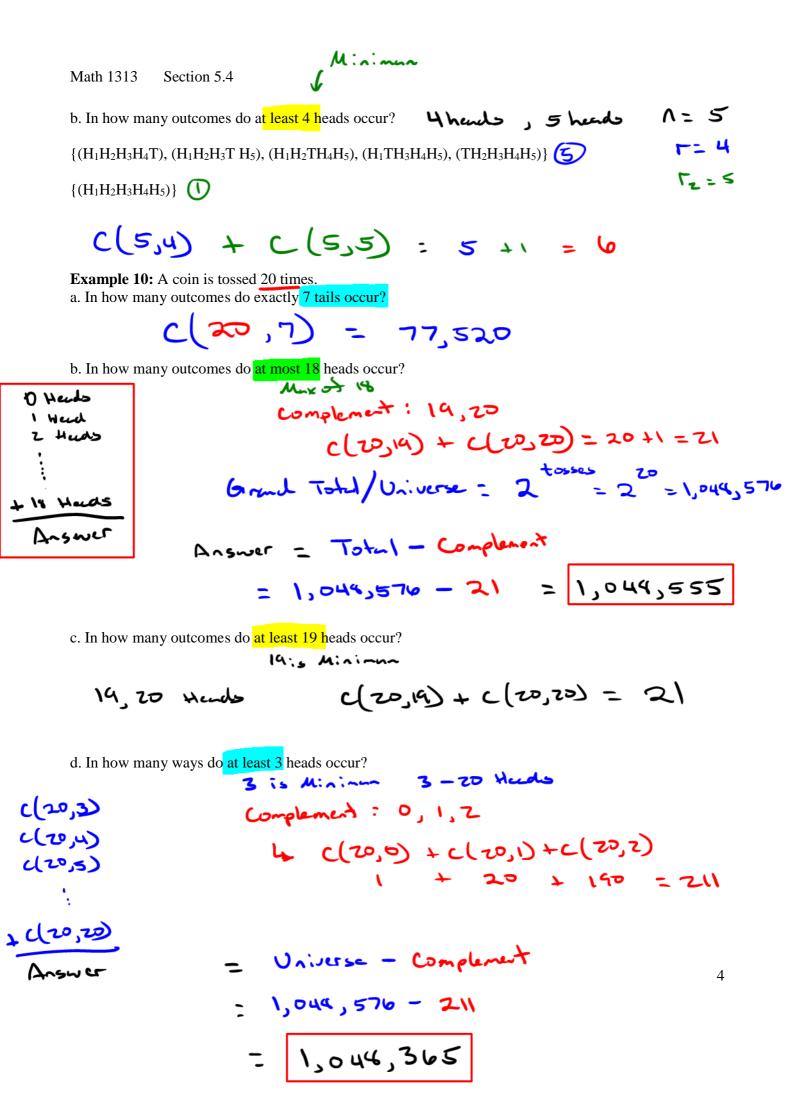
Order Not important ヘニロ C(10,1) = 210 r=4

Example 9: A coin is tossed 5 times. a. In how many outcomes do exactly 3 heads occur?

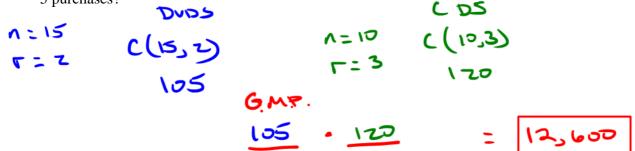
Order of Hends Doesn't Matter

 $\{(H_1H_2H_3TT), (H_1H_2T H_4T), (H_1H_2TT H_5), (H_1TH_3T H_5), (H_1TTH_4H_5), \ (H_1T H_3H_4T), (TH_2H_3H_4T), (TH_2H_3T H_5), (TH_2TH_4H_5), (TTH_3H_4H_5)\} \ 5$

C(5,3) = 10 ハニ ら r= 3 3



Example 11: A student belongs to a entertainment club. This month he must purchase 2 DVDs and 3 CDs. If there are 15 DVDs and 10 CDs to choose from, in how many ways can he choose his 5 purchases?



Example 12: A committee of 16 people, 7 women and 9 men, is forming a subcommittee that is to be made up of 6 women and 6 men. In how many ways can the subcommittee be formed?



Example 13: A computer store receives a shipment of 35 laser printers, including 6 that are defective. Five of these printers are selected to be displayed in the store.

le Des 29 Good

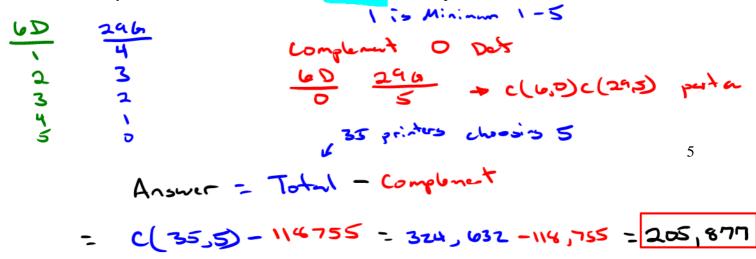
a. How many of these selections will contain no defective printers?

 $\frac{246}{5} = \frac{246}{5} = \frac{2(6,0)}{5} = \frac{246}{5} = \frac$

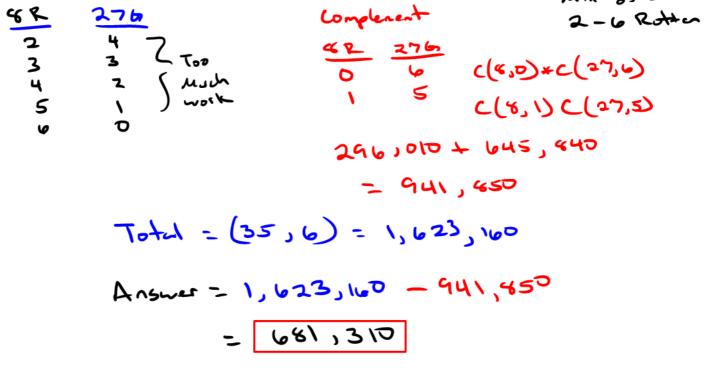
b. How many of these selections will contain 1 defective printer?



c. How many of these selections will contain at least 1 defective printer?



Example 14: A customer at a fruit stand picks a sample of 6 avocados at random from a crate containing 35 avocados of which 8 are rotten. In how many ways can the batch contain at least 2 rotten avocados?



Popper 5: An urn contains 17 red marbles and 18 blue marbles. 16 marbles are chosen. In how many ways can 6 red marbles be chosen?

- a. 102
- b. 541549008
- c. 8008
- d. 56134
- e. 12376