Section 4.3 - Amortization and Sinking Funds

Sinking Fund "Saving"

Is a fund accumulated over time in order to pay off a debt or meet future goals or obligations.

The periodic payment E required to accumulate a sum of F dollars over n periods with interest charged at the rate of i per period is

$$E = \frac{Fi}{(1+i)^n - 1}$$

Amortization "Zeroing out a Balance"

Is the process of paying off a debt with equal periodic payments made over a specified period of time that includes a portion of the principal and interest.

The periodic payment E on a loan of P dollars to be amortized over n periods with interest charged at the rate of i per period is

$$E = \frac{Pi}{1-(1+i)^{-n}}$$

Example 1: Kelly wishes to buy a car that costs $32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 5 years. She decides to secure the loan from the dealer. How much will her monthly payments be? Amortization

$r = 0.0625$
$t = 5$
$m = 12$
$n = 60$
$i = \frac{r}{m} = \frac{0.0625}{12}$

$P = 32,998$

$$E = \frac{Pi}{(1-(1+i)^{-n})}$$

$$= 32998 * 0.0625/12 \Big/ \left(1-(1+0.0625/12)^{-60}\right)$$

$\approx \$641.79$

Example 2: A person would like to have $200,000 in an account for retirement 15 years from now. How much should be deposited quarterly in an account paying 6% per year compounded quarterly to obtain this amount?

"Save" → Sinking Fund

$F = 200{,}000$
$t = 15$
$m = 4$
$r = 0.06$
$n = 60$
$i = \frac{0.06}{4}$

$$E = \frac{F\,i}{((1+i)^n - 1)}$$

$$= 200000 * 0.06/4 \Big/ \left((1+0.06/4)^{60} - 1 \right)$$

$$= \$2,078.69$$

Example 3: A sailboat costs $16,000. You pay 15% down and secure a loan for the remaining balance. How much are your monthly payments if 18% per year compounded monthly is charged over a period of 6 years?

Popper 2: Identify the type of problem.

a. Sinking Fund
b. Amortization

$t = 6$ yrs
$r = 0.18$
$m = 12$
$P = 16000 - DP$
$16000 - 0.15(16000)$
$= 13600$

$$E = \frac{P\,i}{1 - (1+i)^{-n}}$$

$$= 13600 * 0.18/12 \Big/ \left(1 - (1+0.18/12)^{-72} \right)$$

$$= \$310.19$$

Example 4: Christina plans to go to Disney World in two summers and wishes to have $7000 by then. How much money should she deposit monthly for the next 2 years in an account paying 3.25% per year compounded monthly to achieve this goal?

"Save"

$F = 7000$
$r = 0.0325$
$t = 2$
$m = 12$
$n = 24$
$i = \frac{0.0325}{12}$

$$E = \frac{F\,i}{((1+i)^n - 1)}$$

$$= 7000 * 0.0325/12 \Big/ \left((1+0.0325/12)^{24} - 1 \right)$$

$$= \$282.68$$

Example 5: Business partners, Bill and Bob, buy an apartment house for $1,250,000 by making a down payment of $125,000 and financing the rest with semiannual payments over the next 10 years. The interest rate on the debt is 8% per year compounded semiannually. How much is their semiannually payment?

Amortization

$P = 1,250,000 - D.P.$

$= 1125000$

$t = 10$

$m = 2$

$r = 0.08$

$$E = \frac{Pi}{1-(1+i)^{-n}}$$

$$= 1125000 * 0.08/2 \Big/ \left(1 - (1+0.08/2)^{-20}\right)$$

$$= \boxed{82,778.47}$$

Popper 3: The Flores Family loves to go sailing on the weekends. Mr. Flores has decided to purchase a more spacious sailboat. The sailboat he is interested in buying in 3 years will cost him $20,000. An account at Invest Well Bank earns 2% per year compounded monthly. How much should Mr. Flores deposit in this account at the beginning of each month to be able to pay cash for the sailboat in 3 years?

a. 513.24
b. 565.80
c. 539.52
d. 549.52
e. 662.09

Chapter 4: Math of Finance Problems

Identify the type of problem.

1. Anna wants to have $5,000 saved when she graduates from college so that she will have a down payment for a new car. Her credit union pays 5% annual interest compounded monthly. How much money should she deposit each month to have the money available when she graduates in 3 years?

↳ E ?? Amortization or Sinking Fund

2 Bill bought a new car. His financing deal was a 5 year loan at 9.98% annual interest compounded monthly. His monthly payment was $421.25 and he paid no money down. What was the total purchase price of the car?

↳ Annuity

P.V.

P.V. Annuity

3. Sergio wants to have $5,000 in the bank in 3 years to pay for an Alaskan cruise. How much cash should he deposit today, if the bank pays 4% annual interest compounded quarterly, if he wants to be sure to have the funds available in 3 years?

P.V. One time deposit

P.V. w/ Compound Interest

4: Edwin and Frances are buying a new home. The purchase price is $155,000. They will make a 10% down payment on the house. Their loan for the house is a 30 year conventional loan at 6.75% per year compounded monthly. Find their monthly payment. **Identify the type of problem.**

↳ E ??

Amortization or Sinking Fund

5. Grace decides to start a savings program when she gets her first job after graduation. She deposits $2,500 into her credit union savings account. The credit union pays 3.8% annual interest compounded quarterly. How much money will she have in the account after 4 years?

↳ One time Deposit F.V.

F.V. w/ Compound Interest

Chapter 4: Math of Finance Problems

6. Helen bought a new computer. The finance company charged her 15% per year compounded monthly. Her monthly payments were $88.23 for 2 years and she made no down payment. What was the original price of the computer?

↳ E (Annuity)

P.V.

P.V. Annuity

Annuity

7. Gary decided to save some money for his daughter's college education. He decided to save $300 per quarter. His credit union pays 4.5% per year compounded quarterly. How much money will he have available when his daughter starts college in 10 years?

F.V.

F.V. Annuity

8. Jolene owns a clothing store. She anticipates that she will need to replace her telephone system in 3 years. She projects that a new system will cost $12,500. Her bank pays 5% annual interest compounded semiannually. How much should she deposit semiannually in order to be able to pay cash for the new phone system?

↳ E ??

~~Amortization~~ or Sinking Fund

9. Kris wins the lottery and decides to deposit $25,000 of his winnings in an account for his nephew. The bank pays 6.2% annual interest compounded monthly. How much will he be able to give his nephew in 5 years?

F.V.

One time Deposit

F.V. w/ Compound Interest

10. Megan bought a new car. Her car payments are $385.17 for 6 years. Her financing rate was 8.9% annual interest compounded monthly. She made a $1,20[illegible] down payment. What was the total purchase price of the car?

Annuity

↳ Original cost → P.V.

P.V. Annuity

Chapter 4: Math of Finance Problems

11. A company has an immediate need for a loan. In an agreement worked out with its banker, the company assigns its royalty income of $4,800 per month for the next 3 years from certain oil properties to the bank, with the first payment due at the end of the first month. If the bank charges interest at the rate of 9% per year compounded monthly, what is the amount of the loan negotiated between the parties?

12. Carol's employer deposits $1,000 per quarter into a retirement plan that earns 3.5% annual interest compounded quarterly. How much will be in the plan when she retires in 32 years?

13. Kelly wishes to buy a car that costs $32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 5 years. She decides to secure the loan from the dealer. How much will her monthly payments be?

14. David owns a small business and knows that he will need to purchase two new delivery vans in 5 years. He anticipates that the vans will cost the business $28,500 each. His bank pays 4.2% per year compounded monthly. How much should he deposit each month so that he will have the funds available to buy the vans in 5 years?

15. Mary deposited $5,000 in an account that earns 9% per year compounded monthly. How much will she have in 40 years, when she retires?

16. Denise wishes to have $6,000 in an account in 3 years. Her bank will pay 3.25% per year compounded semiannually. How much should she deposit now to have the desired amount of money in the account in 3 years?

Chapter 4: Math of Finance Problems

17. Parents agree to invest $500 at 10% per year compounded semiannually for their son on the December 31 or June 30 following each semester that he makes the Dean's list during his 4 years in college. If he makes the Dean's list in each of the 8 semesters, how much money will his parents have to give him when he graduates in 4 years?

18. A health club offers to let you join for $50 down and payments of only $36 per month for 3 years. When you read the fine print, you discover that the interest rate is 18% per year compounded monthly. What is the cash price of the health club membership? How much will the club membership cost you after 3 years?

19. Nicholas and Olivia are buying a house for $250,000. They made a 15% down payment. Their financing is for 30 years at 6.78% annual interest compounded monthly. Find their monthly payment.

20. A lending company recently offered 36-month auto loans at 7.56% per year compounded monthly to applicants with good credit ratings. If you have a good credit rating and can afford monthly payments of $350, how much can you borrow from the company?

Chapter 4: Math of Finance Problems

Solutions

1. Sinking Fund; $129.02
2. Present Value of an Annuity; $19,835.47
3. Present Value with compound interest; $4,437.25
4. Amortization; $904.79
5. Future Value with compound interest; $2,908.31
6. Present Value of an Annuity; $2,084.79
7. Future Value of an Annuity; $15,050.05
8. Sinking Fund; $1,956.87
9. Future Value with compound interest; $34,058.44
10. Present Value of an Annuity; $22,626.97
11. Present Value of an Annuity; $150,944.67
12. Future Value of an Annuity; $234,281.12
13. Amortization; $641.79
14. Sinking Fund; $855.39
15. Future Value with compound interest; $180,549.51
16. Present Value with compound interest; $5,446.88
17. Future Value of an Annuity; $3,231.61
18. Present Value of an Annuity; $1,045.78; $1,346.00
19. Amortization; $1,382.51
20. Present Value of an Annuity; $11,241.81

Popper 5: A certain company has purchased new swivel chairs for its employees. The company made the purchase on a credit plan at Buy Right. Their monthly payments are $1,000 for 3 years. Buy Right will charge 2.25% per year compounded monthly. How much was the original total price of the furniture? **Identify this type of problem.**

a. Future Value with compound interest
b. Amortization
c. Future Value of an Annuity
d. Sinking Fund
e. Present Value of an Annuity

Math 1313 Section 5.1

Section 5.1 – Sets and Set Operations

A collection of objects is called a **set**.

An object of a set is called an **element**.

Notation:
$\in$ = "element of"
$\notin$ = "not an element of"

Example 1: Let B = {a, b, c, …, y, z}. In **set-builder notation,** the set B can be written as follows:

Set = { variable | condition/criteria }

B = { γ | $\gamma \in$ the alphabet }

Equality of Sets
Let A and B be two sets. We say that A is equal to B, written as A = B. This is true if and only if A and B have exactly the same elements. If two sets are not equal we write A ≠B.

Subsets
Let A and B be two sets. We say that A is a subset of B or that is contained in B and written $A \subseteq B$.
From the definition it follows that for any set A, $A \subseteq A$; that is, every set is a subset of itself.

Proper Subsets
If $A \subseteq B$, but $A \neq B$ then A is a **proper subset** of B. If A is a proper subset of B then we write $A \subset B$
In other words: A is a proper subset of B if the following two conditions hold.

1. $A \subseteq B$
2. There exist at least one element in B that is not in A.

Example 2: Let A ={1,2,3}, B = {1,2,3,4,5}and C = {3,2,1} . In the following, answer true or false in the following:

A = C	T or F
$A \subseteq C$	T or F
$A \subset B$	T or F
$C \subset A$	T or F
$5 \notin C$	T or F

A set that contains no elements is called the **Empty Set**
Note: We write Ø to denote the empty set. The symbol Ø is a subset of every set.

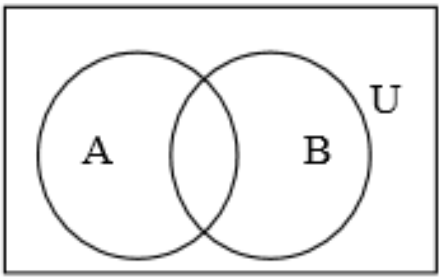
Example 3: Let A = {a, b, c}. List all subsets and proper subsets of the set A.

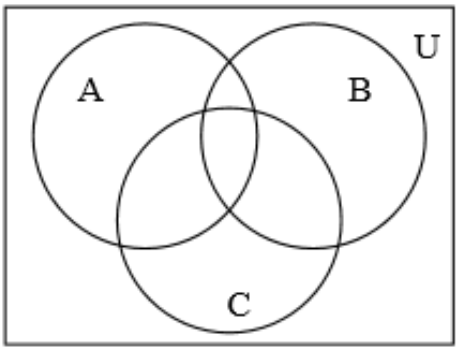
Subsets: $\{\emptyset\}$
$\{a\}, \{b\}, \{c\}$
$\{a,b\}, \{b,c\}, \{a,c\}$
$\{a,b,c\}$

Proper subsets (strictly smaller)
$\{\emptyset\}, \{a\}, \{b\}, \{c\}$
$\{ab\}, \{bc\}, \{a,c\}$

The **Universal set** is the set of interest in a particular discussion.

A **Venn diagram** is a visual representation of sets.
They look like:



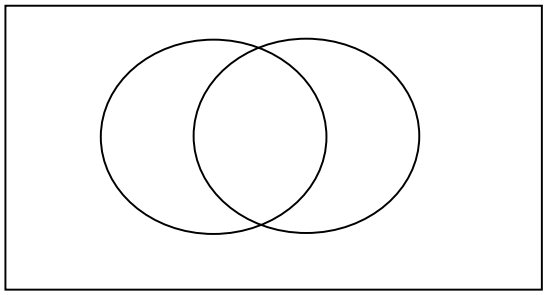


Set Operations

Let A and B be two sets. The set of all elements that that belong to either A *or* B or both is called the **Union** of A and B (denoted A ∪ B).

In set builder notation $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$

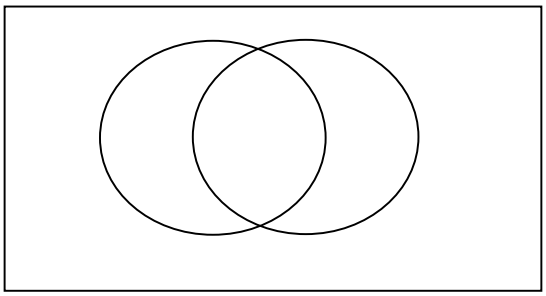
Set Union in a Venn diagram looks like:



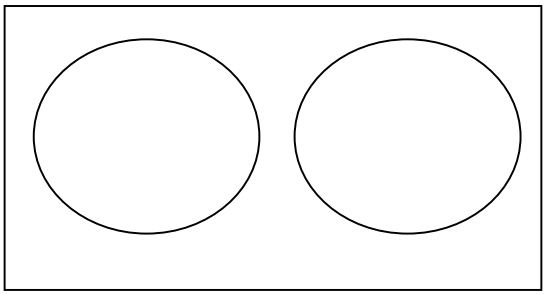
Let A and B be two sets. The set of all elements in common with both sets A *and* B is called the **Intersection** of A and B (denoted $A \cap B$).

In **set-builder notation** $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Set Intersection in a Venn diagram looks like:



If $A \cap B = \emptyset$, then we say the intersection is the **null intersection** and that A and B are **disjoint**.



Let U be a universal set and $A \subseteq U$. The set of all elements in U that are not in A is called the

Complement of A. (denoted A^C)

In **set-builder notation** $A^c = \{x \mid x \in U, x \notin A\}$

Set Complementation in a Venn diagram looks like: