A B
$(m \times n)(n \times p)$
size of Answer

Math 1313 Section 3.4
**Section 3.4: Matrix Multiplication**

If A is a matrix of size m x n and B is a matrix of size n x p then the product AB is defined and is a matrix of size m x p.

So, two matrices can be multiplied if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

**Example 1:** Multiple the given matrices.

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is a 1 x 3 matrix $\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ is a 3 x 1 matrix

$(1 \times 3)(3 \times 1)$

When multiplied the ending matrix will be 1 x 1.

rows times column

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

$(1 \cdot 6 + 2 \cdot 5 + 3 \cdot 4)$

$(6 + 10 + 12) = [28]$

Here is how you multiply:

$(2 \times 2)\ (2 \times 1) = 2 \times 1$

$R_1 C_1$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \begin{bmatrix} a_{11} \times b_{11} + a_{12} \times b_{21} \\ a_{21} \times b_{11} + a_{22} \times b_{21} \end{bmatrix}$$

$R_2 \cdot C_1$

**Example 2:** Multiply the given matrices.

**a.** $\begin{bmatrix} -2 & 4 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -3 \\ 5 \end{bmatrix}$

$(2 \times 2)(2 \times 1)$

$= 2 \times 1$

$R_1 \cdot C_1$
$(-2)(-3) + 4(5)$
$R_2 \cdot C_1$
$1(-3) + 0(5)$

$= \begin{matrix} 6 + 20 \\ -3 + 0 \end{matrix} = \begin{pmatrix} 26 \\ -3 \end{pmatrix}$

**b.** $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 8 \\ 6 \end{bmatrix}$

$(2 \times 3)(3 \times 1)$

$= 2 \times 1$

$R_1 \cdot C_1$
$2(1) + 3(8) - 1(6)$
$R_2 \cdot C_1$
$4(1) + 2(8) + 2(6)$

$= \begin{pmatrix} 2 + 24 - 6 \\ 4 + 16 + 12 \end{pmatrix} = \begin{pmatrix} 20 \\ 32 \end{pmatrix}$

From Popper 7

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Question **5:** The following represent matrices and the dimension of each is also stated below.

A is a matrix of size 4X2
B is a matrix of size 3X5
C is a matrix of size 7X2
D is a matrix of size 5X4
E is a matrix of size 5X3

The product BD is defined.

a. True
b. False

**Example 3:** Mike and Sam have stock as follows:

$$\mathbf{A} = \begin{array}{c} \begin{array}{cccc} \text{BAC} & \text{GM} & \text{IBM} & \text{TRW} \end{array} \\ \begin{bmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{bmatrix} \end{array}$$

$2 \times 4$

Mike is this row one and Sam row two

At the close of trading on a certain day, the price $/share (GM, IBM, BAC, respectively) are:

$4 \times 1$

$$\mathbf{B} = \begin{bmatrix} 54 \\ 48 \\ 98 \\ 82 \end{bmatrix}$$

$$\begin{pmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{pmatrix} \begin{pmatrix} 54 \\ 48 \\ 98 \\ 42 \end{pmatrix}$$

$$100 \begin{pmatrix} 2 & 3 & 1 & 2 \\ 1 & 2 & 4 & 0 \end{pmatrix} \begin{pmatrix} 54 \\ 48 \\ 98 \\ 42 \end{pmatrix}$$

**AB =**

$$100 \begin{pmatrix} 2(54) + 3(48) + 1(98) + 2(82) \\ 1(54) + 2(48) + 4(98) + 0(82) \end{pmatrix}$$

$$100 \begin{pmatrix} 108 + 144 + 98 + 164 \\ 54 + 96 + 392 + 0 \end{pmatrix}$$

$$= 100 \begin{pmatrix} 514 \\ 542 \end{pmatrix} = \begin{matrix} \$51400 & \text{Mike} \\ \$54200 & \text{Sam} \end{matrix}$$

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**Example 4:** Multiply the following matrices if possible.

Let $A = \underset{2\times 3}{\begin{pmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{pmatrix}}$, $B = \underset{3\times 3}{\begin{pmatrix} 3 & 1 & 4 \\ 2 & 0 & 3 \\ 1 & 2 & -1 \end{pmatrix}}$, $C = \underset{2\times 2}{\begin{pmatrix} -10 & 9 \\ -6 & 4 \end{pmatrix}}$, and $D = \underset{4\times 2}{\begin{pmatrix} -3 & 9 \\ 6 & 1 \\ 0 & 9 \\ 8 & 4 \end{pmatrix}}$ compute, if possible:

AB

$(2\times 3)(3\times 3)$

$= 2\times 3$

$$\begin{pmatrix} \overset{R_1\cdot C_1}{1(3)+3(2)+0(1)} & \overset{R_1\cdot C_2}{1(1)+3(0)+0(2)} & \overset{R_1\cdot C_3}{1(4)+3(3)+0(-1)} \\ \overset{R_2\cdot C_1}{2(3)+4(2)-1(1)} & \overset{R_2\cdot C_2}{2(1)+4(0)-1(2)} & \overset{R_2\cdot C_3}{2(4)+4(3)-1(-1)} \end{pmatrix}$$

$$\begin{pmatrix} 9 & 1 & 13 \\ 13 & 0 & 21 \end{pmatrix}$$

CD $= (2\times 2)(4\times 2)$

Don't Match

CD is NOT Possible

CA $(2\times 2)(2\times 3) = 2\times 3$

$$\begin{pmatrix} -10 & 9 \\ -6 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{pmatrix}$$

| $R_1\cdot C_1$ | $R_1\cdot C_2$ | $R_1\cdot C_3$ |
| --- | --- | --- |
| $-10+18$ | $-30+36$ | $0-9$ |
| $R_2\cdot C_1$ | $R_2\cdot C_2$ | $R_2\cdot C_3$ |
| $-6+8$ | $-18+16$ | $0-4$ |

$$= \begin{pmatrix} 8 & 6 & -9 \\ 2 & -2 & -4 \end{pmatrix}$$

**Laws for Matrix Multiplication**

If the products and sums are defined for the matrices A, B and C, then

1. (AB)C = A(BC)
2. A(B + C) = AB + AC

Note: In general, matrix multiplication is not commutative – that is, AB $\neq$ BA.

Order of Multiplication is Important

**Example 5**: If A and B are matrices we will look at the product AB and BA.

$$A \underset{}{\overset{2\times 2}{=}} \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix} \qquad B \overset{2\times 2}{=} \begin{bmatrix} -1 & 2 \\ 5 & 7 \end{bmatrix}$$

AB =

$R_1 \cdot C_1$ $\quad$ $R_1 \cdot C_2$

$3+20$ $\quad$ $-6+28$

$R_2 \cdot C_1$ $\quad$ $R_2 \cdot C_2$

$-2+0$ $\quad$ $4+0$

$$= \overset{AB}{\begin{pmatrix} 23 & 22 \\ -2 & 4 \end{pmatrix}}$$

$$BA = \begin{bmatrix} -1 & 2 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix}$$

$R_1 \cdot C_1$ $\quad$ $R_1 \cdot C_2$

$3+4$ $\quad$ $-4+0$

$R_2 \cdot C_1$ $\quad$ $R_2 \cdot C_2$

$-15+14$ $\quad$ $20+0$

$$= \overset{BA}{\begin{pmatrix} 7 & -4 \\ -1 & 20 \end{pmatrix}}$$

## Identity Matrix

The square matrix of size n having 1s along the main diagonal and zeros elsewhere is called the identity matrix of size n.

The **identity matrix** of size n is given by
$$I_n = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & 1 \end{pmatrix}$$

$I_1 = [1]$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

If A is a square matrix of size n, then $I_n A = AI_n = A$.

"One Concept" for Matrices

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**Example 6:** Given the following matrices,

$$X = \begin{pmatrix} 0 & 1 & -2 \\ 4 & -2 & 1 \\ 5 & 0 & -3 \end{pmatrix}, \qquad Y = \begin{pmatrix} 2 & 3 & -4 & 1 \\ -5 & 2 & 1 & 6 \\ 0 & -2 & 3 & -4 \end{pmatrix}$$

$$(3 \times 3) \qquad\qquad (3 \times 4)$$

a. Is XY defined, if so what is the size?

Yes ; $(3\times3)(3\times4) = \boxed{(3\times4)}$

b. Let A=XY, what is $a_{23}$?

→ The element in 2nd row 3rd column

$4(-4) + (-2)(1) + 1(3)$

$-16 - 2 + 3$

$= \boxed{-15}$

Question

**[redacted] 4:** Given the following matrices, Let X = AB.

$$\underset{A}{\begin{pmatrix} \mathbf{-2} & \mathbf{4} & \mathbf{5} & \mathbf{0} \\ \mathbf{1} & \mathbf{-3} & \mathbf{2} & \mathbf{3} \end{pmatrix}} \underset{B}{\begin{pmatrix} \mathbf{9} & \mathbf{3} & \mathbf{2} \\ \mathbf{-7} & \mathbf{-1} & \mathbf{3} \\ \mathbf{5} & \mathbf{0} & \mathbf{-2} \\ \mathbf{1} & \mathbf{2} & \mathbf{6} \end{pmatrix}}$$

**Find $x_{2,2}$.**

a. 12
b. 6
c. 43
d. 7

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**Example 7:** The following table displays the average grade in each category for an upper level honors course with 4 students.

| | Test 1 | Test 2 | Test 3 | Final Exam | Homework Avg | Quiz Avg |
|---|---|---|---|---|---|---|
| Mark | 94 | 80 | 78 | 86 | 91 | 92 |
| Ashley | 80 | 88 | 90 | 85 | 76 | 100 |
| Scott | 100 | 75 | 88 | 82 | 84 | 88 |
| Melissa | 70 | 82 | 86 | 90 | 78 | 91 |

If each test is worth 16%, the final exam is worth 24%, the homework average is worth 12%, and the quiz average is worth 16%, what is each student's course average? Use a matrix to display the grades and another to display the percentages. Give the answer in the form of a matrix.

$$\underset{(4 \times 6)}{\begin{pmatrix} 94 & 80 & 78 & 86 & 91 & 92 \\ 80 & 88 & 90 & 85 & 76 & 100 \\ 100 & 75 & 88 & 82 & 84 & 88 \\ 70 & 82 & 86 & 90 & 78 & 91 \end{pmatrix}} \underset{6 \times 1}{\begin{pmatrix} 0.16 \\ 0.16 \\ 0.16 \\ 0.24 \\ 0.12 \\ 0.16 \end{pmatrix}} = \underset{4 \times 1}{\begin{pmatrix} 86.6 \\ 86.8 \\ 85.92 \\ 83.6 \end{pmatrix}}$$

Question 3

[redacted]: The following represent matrices and the dimension of each is also stated below.

A is a matrix of size 4X2
B is a matrix of size 3X5
C is a matrix of size 7X2
D is a matrix of size 5X4
E is a matrix of size 5X3

The product CA is defined.

a. True
b. False

$$\frac{4}{7} \cdot \frac{7}{4} = 1$$

Math 1313 Section 3.5

**Section 3.5: The Inverse of a Matrix**

Over the set of real number we have what we call the **multiplicative inverse** or **reciprocal**. The multiplicative inverse of a number is a second number that when multiplied by the first number yields the **multiplicative identity 1**.

This is where the Identity Matrix comes in.

Let A be a square matrix of size n and another square matrix $A^{-1}$ of size n such that $AA^{-1} = A^{-1}A = I_n$ is called the **inverse of A**.

Note: Not every square matrix has an inverse. A matrix with no inverse is called **singular**.

**Finding the Inverse of a Matrix**

Given the n x n matrix *A*:

1. Adjoin the n x n identity matrix *I* to obtain the augmented matrix $(A \mid I)$

2. Use the Gauss-Jordan elimination method to reduce $(A \mid I)$ to the form $(I \mid B)$, if possible.

The matrix *B* is the inverse of *A*.

**Example 1:** Find the inverse, if possible and check:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right)$$

$R_1 + R_2 \to R_2$

$$\begin{array}{cccc} 1 & 2 & 1 & 0 \\ -1 & 3 & 0 & 1 \\ \hline 0 & 5 & 1 & 1 \end{array}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 5 & 1 & 1 \end{array}\right)$$

$\frac{1}{5} R_2 \to R_2$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} \end{array}\right)$$

$-2 R_2 + R_1 \to R_1$

$$\begin{array}{cccc} 0 & -2 & -\frac{2}{5} & -\frac{2}{5} \\ 1 & 2 & 1 & 0 \\ \hline 1 & 0 & \frac{3}{5} & -\frac{2}{5} \end{array}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & -\frac{2}{5} \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$$

**Example 2:** Find the inverse of a 3 x 3 matrix.(Use Gauss-Jordan)

$$C = \begin{pmatrix} 1 & 4 & -1 \\ 2 & 3 & -2 \\ -1 & 2 & 3 \end{pmatrix}$$

**Example 3:** Find the inverse.

$$B = \begin{pmatrix} 4 & 2 & 2 \\ -1 & -3 & 4 \\ 3 & -1 & 6 \end{pmatrix}$$

**Matrices That Have No Inverses**

If there is a row to the left of the vertical line in the augmented matrix containing all zeros, then the matrix does not have an inverse. Example 3 has this problem and does not have an inverse.