Math 1313 Section 3.2

Example 11: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$3x + y - 4z = 6$$
$$-15x - 5y + 20z = -36$$

More Variables than Equations

a. No Solution

b. Infinitely Many

$$\left(\begin{array}{ccc|c} 3 & 1 & -4 & 6 \\ -15 & -5 & 20 & -36 \end{array}\right)$$

x·5 x·5 x·5 x·6

$5R_1 + R_3 \to R_3$

$$\begin{array}{cccc} 15 & 5 & -20 & 30 \\ -15 & -5 & 20 & -36 \\ \hline 0 & 0 & 0 & -6 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 3 & 1 & -4 & 6 \\ 0 & 0 & 0 & \text{Not } 0 \end{array}\right)$$

Example 12: Solve the system of linear equations using the Gauss-Jordan elimination method.

$$2x - 3y = 13$$
$$x + y = -1$$
$$x - 4y = 14$$

More Equation than Variables

a One solution

b No solution

c Infinitely Many

$$\left(\begin{array}{cc|c} 2 & -3 & 13 \\ 1 & 1 & -1 \\ 1 & -4 & 14 \end{array}\right) \quad R_1 \leftrightarrow R_2 \quad \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 2 & -3 & 13 \\ 1 & -4 & 14 \end{array}\right)$$

$-2R_1 + R_2 \to R_2$

$$\begin{array}{ccc} -2 & -2 & 2 \\ 2 & -3 & 13 \\ \hline 0 & -5 & 15 \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -5 & 15 \\ 1 & -4 & 14 \end{array}\right)$$

$-R_1 + R_3 \to R_3$

$$\begin{array}{ccc} -1 & -1 & 1 \\ 1 & -4 & 14 \\ \hline 0 & -5 & 15 \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -5 & 15 \\ 0 & -5 & 15 \end{array}\right) \xrightarrow{-R_2 + R_3 \to R_3} \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -5 & 15 \\ 0 & 0 & 0 \end{array}\right)$$

$-\frac{1}{5} R_2 \to R_2$

$$\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right)$$

→0

$-R_2 + R_1 \to R_1$

$$\begin{array}{ccc} 0 & -1 & 3 \\ 1 & 1 & -1 \\ \hline 1 & 0 & 2 \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right)$$

$0 = 0$

$$\boxed{\begin{array}{l} x = 2 \\ y = -3 \end{array}}$$

Question 1: When is Quiz 1 due?

a. February 6
b. February 7
c. February 8
d. At the end of the semester

Math 1313 Section 3.3

Section 3.3: Matrix Operations

Addition and Subtraction of Matrices

If A and B are two matrices of the same size,

1. A + B is the matrix obtained by adding the corresponding entries in the two matrices.

2. A – B is the matrix obtained by subtracting the corresponding entries in B from A.

Laws for Matrix Addition

If A, B, and C are matrices of the same dimension, then

1. A + B = B + A

2. (A + B) + C = A + (B + C)

Example 1: Refer to the following matrices: If possible,

$$\underset{3\times 3}{A = \begin{bmatrix} 8 & -3 & 1 \\ 0 & -9 & -4 \\ 9 & 6 & 7 \end{bmatrix}}, \quad \underset{3\times 3}{B = \begin{bmatrix} -5 & 4 & -1 \\ 8 & 4 & 8 \\ 10 & 15 & -2 \end{bmatrix}}, \quad \underset{2\times 3}{C = \begin{bmatrix} 10 & -8 & 3 \\ 5 & -4 & 2 \end{bmatrix}}, \quad \underset{2\times 3}{D = \begin{bmatrix} 4 & 1 & 3 \\ 8 & 5 & 1 \end{bmatrix}}$$

a. compute A – B

$$\begin{matrix} 8-(-5) & -3-4 & 1-(-1) \\ 0-8 & -9-4 & -4-8 \\ 9-10 & 6-15 & 7-(-2) \end{matrix} = \begin{pmatrix} 13 & -7 & 2 \\ -8 & -13 & -12 \\ -1 & -9 & 9 \end{pmatrix}$$

b. compute B + C. Not possible ; Different Sizes

c. compute D + C.

$$\begin{pmatrix} 4+10 & 1-8 & 3+3 \\ 8+5 & 5-4 & 1+2 \end{pmatrix} = \begin{pmatrix} 14 & -7 & 6 \\ 13 & 1 & 3 \end{pmatrix}$$

Scalar Multiplication

A **scalar** is a real number.

Scalar multiplication is the product of a scalar and a matrix. To perform scalar multiplication, each element in the matrix is multiplied by the scalar; hence, it "scales" the elements in the matrix

Example 2: Let $\underset{2\times 2}{A=\begin{pmatrix}1&2\\3&4\end{pmatrix}}$, $\underset{2\times 2}{B=\begin{pmatrix}-1&4\\-7&9\end{pmatrix}}$, and $\underset{2\times 3}{C=\begin{pmatrix}1&2&3\\-6&-9&1\end{pmatrix}}$ find, if possible,

a. -3C $\quad -3\begin{pmatrix}1&2&3\\-6&-9&1\end{pmatrix} = \boxed{\begin{pmatrix}-3&-6&-9\\18&27&-3\end{pmatrix}}$

b. -2B – A $\quad -2\begin{pmatrix}-1&4\\-7&9\end{pmatrix} - \begin{pmatrix}1&2\\3&4\end{pmatrix}$

$$\begin{pmatrix}2&-8\\14&-18\end{pmatrix} - \begin{pmatrix}1&2\\3&4\end{pmatrix} = \boxed{\begin{pmatrix}1&-10\\11&-22\end{pmatrix}}$$

c. 3B + 2C $\quad$ Not possible; Different sizes

Transpose of a Matrix $\quad$ rows $\to$ columns $\quad$ columns $\to$ rows

If A is an m x n matrix with elements a_{ij}, then the **transpose** of A is the n x m matrix A^T with elements a_{ji}.

$$A = \begin{bmatrix}2&5&50\\1&3&27\\16&45&1\end{bmatrix} \qquad A^T = \begin{bmatrix}2&1&16\\5&3&45\\50&27&1\end{bmatrix}$$

Example 3: Given the following matrices, find their transpose.

a. $B=\begin{pmatrix}-3&0&6\\10&100&3\end{pmatrix}$

2x3

$$\underset{3\times 2}{B^T} = \begin{pmatrix}-3&10\\0&100\\6&3\end{pmatrix}$$

b. $D = \begin{pmatrix} 0 \\ -4 \\ 11 \\ -3 \end{pmatrix}$

$D^T = (0 \ -4 \ 11 \ -3)$

Equality of Matrices

Two matrices are equal if they have the same dimension and their corresponding entries are equal.

Example 4: Solve the following matrix equation for w, x, y, and z.

$$\begin{bmatrix} w+6 & x \\ y-2 & z \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$$

$w+6=-2$

$w=-8$

$x=0$

$y-2=1$

$y=3$

$z=4$

Example 5: Solve for the variables in the matrix equation.

$$-\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} + 9\begin{bmatrix} u-6 & 2z+5 \\ y & -\frac{1}{3} \end{bmatrix} = -2\begin{bmatrix} 3 & -8 \\ 1 & v \end{bmatrix}$$

$$\begin{pmatrix} -1 & 2 \\ -4 & -3 \end{pmatrix} + \begin{pmatrix} 9u-54 & 18z+45 \\ 9y & -3 \end{pmatrix} = \begin{pmatrix} -6 & 16 \\ -2 & -2v \end{pmatrix}$$

$-1+9u-54=-6$

$9u-55=-6$

$9u=49$

$u=\frac{49}{9}$

$2+18z+45=16$

$18z+47=16$

$18z=-31$

$z=\frac{-31}{18}$

$-4+9y=-2$

$9y=2$

$y=\frac{2}{9}$

$-3-3=-2v$

$-6=-2v$

$3=v$

Math 1313 Section 3.3

Use the following matrices

$$A = \begin{pmatrix} -2 & 3 & 1 \\ 7 & -4 & -5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 8 \\ 3 & -4 \\ -7 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & -2 & 3 \\ -4 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} -2 & -5 \\ -3 & 7 \\ 8 & -1 \end{pmatrix}$$

Popper 3: Is $3A - 2C$ possible?

a. Yes
b. No

Popper 5: Let $X = -4B + 3D$. Identify x_{21}

a. -15
b. 3
c. 0
d. -21
e. None of the above