Example 3: The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs $1,200 to rent. Each van can transport 8 students, requires 1 chaperone, and cost $100 to rent. The officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. How many vehicles of each type should the officers rent in order to minimize the transportation costs? What are the minimal transportation costs?

a. Define your variables.

x - # of Buses y - # of Vans

b. Construct and fill in a table.

	x	y		
students	40	8	$\geq$	400
Chaperones	3	1	$\leq$	36
Cost	1200	100		

c. State the Linear Programming Problem. Do Not Solve.

Min $C = 1200x + 100y$

s.t.

$$40x + 8y \geq 400$$

$$3x + y \leq 36$$

$$x, y \geq 0$$

Example 4: A 4-H member raises only geese and pigs. She wants to raise no more than 16 animals. It costs her $ 500 to raise a goose and $ 1500 to raise a pig. And she has $ 18,000 available for the project. The 4-H member wishes to maximize her profit. Each goose produces $ 600 in the profit and each pig $ 2000 profit. How many of each animal should she raise to maximize her profit?

a. Define your variables.

x - # of geese   y - # of pigs

b. Construct and fill in a table.

	x	y		
Animals	1	1	≤	16
Money	500	1500	≤	18000
Profit	600	2000		

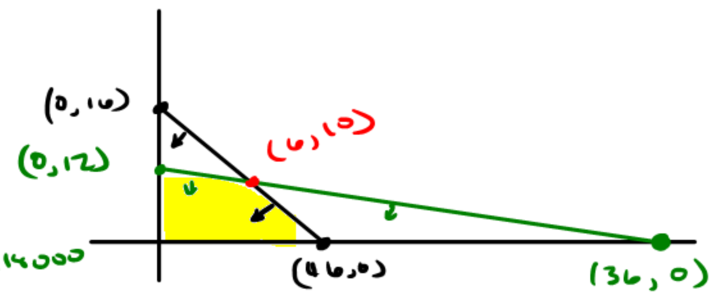
c. State the Linear Programming Problem and Solve   Ex 4 from 2.1

Max Profit $= 600x + 2000y$

s.t. $x + y \le 16$

$500x + 1500y \le 18000$

$x, y \ge 0$



$x + y \le 16$

$y \le -x + 16$

$500x + 1500y \le 18000$

$5x + 15y \le 180$

$15y \le -5x + 180$

$y \le -\frac{1}{3}x + 12$

$-x + 16 = -\frac{1}{3}x + 12$

$+\frac{1}{3}x \quad -16 \qquad +\frac{1}{3}x \quad -16$

$-\frac{2}{3}x = -4$

$x = 6$

$y = 10$

Pt	Max $P = 600x + 2000y$
(0,12)	$600(0) + 2000(12) = 24000$
(6,10)	$600(6) + 2000(10) = 23{,}600$
(16,0)	$600(16) + 2000(0) = 9{,}600$
(0,0)	0

$\boxed{\text{Raise 0 geese \& 12 pigs for a profit of \$24000}}$

rows by columns
L,R UP

Section 3.1: Matrices

A **matrix** is an ordered rectangular array of numbers, letters, symbols or algebraic expressions. A matrix with m rows and n columns has **size** or **dimension** m x n.

The real numbers that make up the matrix are called **entries** or **elements** of the matrix. The entry in the *i*th row and *j*th column is denoted by a_{ij}

A matrix with only one column or one row is called a **column matrix** (or **column vector**) or **row matrix** (or **row vector**), respectively.

Example 1: Given $A = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{pmatrix} 2 & 7 & 7 \\ -5 & 3 & 9 \\ 0 & -10 & 20 \\ 1 & -3 & -11 \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$,

a. what is the dimension of *A*? 4 rows × 3 columns $\boxed{4 \times 3}$

b. identify a_{43}. 4th row, 3rd Column $= \boxed{-11}$

Systems of Linear Equations in Matrix Form

In order to write a system of linear equations in matrix form, first make sure the like variables occur in the same column. Then we'll leave out the variables of the system and simply use the coefficients and constants to write the matrix form.

Given the following system of equations:

$$\begin{aligned} 2x + 4y + 6z &= 22 \\ 3x + 8y + 5z &= 27 \\ -x + y + 2z &= 2 \end{aligned}$$

The **coefficient matrix** is: $\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} 2 & 4 & 6 \\ 3 & 8 & 5 \\ -1 & 1 & 2 \end{pmatrix} \end{array}$

The **constant matrix** is: $\begin{pmatrix} 22 \\ 27 \\ 2 \end{pmatrix}$

Dashed/Solid = "Equal Sign"

The **augmented matrix** is: $\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 2 & 4 & 6 & 22 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array}\right) \end{array}$

Example 2: Give the coefficient, constant and augmented matrix for the system of equations.

$$2x - 4y = 15$$
$$-3y + 2z = 9$$
$$x + y - 3z = -8$$

Coefficient $= \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} 2 & -4 & 0 \\ 0 & -3 & 2 \\ 1 & 1 & -3 \end{pmatrix} \end{array}$

constant $= \begin{pmatrix} 15 \\ 9 \\ -8 \end{pmatrix}$

Augmented $\left(\begin{array}{ccc|c} 2 & -4 & 0 & 15 \\ 0 & -3 & 2 & 9 \\ 1 & 1 & -3 & -8 \end{array}\right)$

Popper 4

Question 2: Give the following matrix, identify $s_{3,2}$

$$S = \begin{pmatrix} -7 & 2 & -4 & 4 \\ 0 & 1 & 3 & -3 \\ -1 & 5 & -2 & 2 \\ 8 & 7 & 0 & 6 \end{pmatrix}$$

a. 3
b. -3
c. 4
d. 5
e. None of the Above

Math 1313 Section 3.2

Section 3.2: Solving Systems of Linear Equations Using Matrices

As you may recall from College Algebra or Section 1.3, you can solve a system of linear equations in two variables easily by applying the substitution or addition method. Since these methods become tedious when solving a large system of equations, a suitable technique for solving such systems of linear equations will consist of Row Operations. The sequence of operations on a system of linear equations are referred to equivalent systems, which have the same solution set.

Row Operations

1. Interchange any two rows.

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & 5 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 3 \end{bmatrix}$$

2. Replace any row by a nonzero constant multiple of itself. Create a 1

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 8 \end{bmatrix} \quad \frac{1}{4}R_2 \to R_2 \quad \begin{bmatrix} 2 & -1 & 3 \\ 1 & -\frac{1}{2} & 2 \end{bmatrix}$$

3. Replace any row by the sum of that row and a constant multiple of any other row. Create a 0

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right] \quad -2R_1 + R_2 \to R_2 \quad \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \end{array}\right]$$

$-2(1\ 3\ 5) \Rightarrow$

$$\begin{array}{rrr} & -2 & -6 & -10 \\ + & 2 & -1 & 3 \\ \hline & \underbrace{0 \quad -7 \quad -7}_{\text{New } R_2} \end{array}$$

Row Reduced Form

An m x n augmented matrix is in row-reduced form if it satisfies the following conditions:

1. Each row consisting entirely of zeros lies below any other row having nonzero entries.

$$\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right] \quad \underset{\text{the correct row-reduced form}}{R_2 \leftrightarrow R_3} \quad \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

2. The first nonzero entry in each row is 1 (called a leading 1).

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right] \quad \underset{\text{the correct row-reduced form}}{\frac{1}{2}R_2 \to R_2} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

3. If a column contains a leading 1, then the other entries in that column are zeros.

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$ the correct row-reduced form $$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$2R_2 + R_1 \to R_1$

0 0 2 -2
1 2 -2 3
1 2 0 1

4. In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row.

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|c} 0 & 1 & -2 \\ 1 & 0 & 3 \end{array}\right]$$ the correct row-reduced form $$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$$

Example 1: Determine which of the following matrices are in row-reduced form. If a matrix is not in row-reduced form, state which condition is violated.

a. $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right)$ Yes

d. $\left(\begin{array}{ccc|c} 1 & -9 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Yes

b. $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ NO
2 should be zero

e. $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 2 & 6 \end{array}\right)$ NO; Needs to be a one

$\to 1$

c. $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right)$ NO
Row full of zeros, should be at Bottom

f. $\left(\begin{array}{cc|c} 0 & 1 & 1 \\ 1 & 0 & 5 \end{array}\right)$ NO; wrong direction

Math 1313 Section 3.2

Popper 4

Question 4: Is the following matrix in row reduced form.

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

a. Yes
b. No

The Gauss-Jordan Elimination Method

1. Write the augmented matrix corresponding to the linear system.

2. Use row operations to write the augmented matrix in row reduced form. If at any point a row in the matrix contains zeros to the left of the vertical line and a nonzero number to its right, stop the process, as the problem has no solution.

3. Read off the solution(s).

There are three types of possibilities after doing this process.

Unique Solution

Example 2: The following augmented matrix in row-reduced form is equivalent to the augmented matrix of a certain system of linear equations. Use this result to solve the system of equations.

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \end{array}\right)$$

$x + 0y = 4$
$0x + y = -2$;

$$\boxed{\begin{aligned} x &= 4 \\ y &= -2 \end{aligned}}$$

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$x + 0 + 0 = 2$
$0 + y + 0 = -7$
$0 + 0 + z = 3$

$$\boxed{\begin{aligned} x &= 2 \\ y &= -7 \\ z &= 3 \end{aligned}}$$

Example 3: Solve the system of linear equations using the Gauss-Jordan elimination method.

$x + 2y = 1$
$2x + 3y = -1$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 3 & -1 \end{array}\right)$$

$\to 0$

$-2R_1 + R_2 \to R_2$

-2	-4	-2
2	3	-1
0	-1	-3

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -1 & -3 \end{array}\right)$$

$\to 1$

$-1 \cdot R_2 \to R_2$

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 3 \end{array}\right)$$

$-2R_2 + R_1 \to R_1$

0	-2	-6
1	2	1
1	0	-5

$$\left(\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right)$$

$$\boxed{\begin{array}{l} x = -5 \\ y = 3 \end{array}}$$

Example 4: Solve the system of linear equations using the Gauss-Jordan elimination method.

$3x + y = 1$
$-7x - 2y = -1$