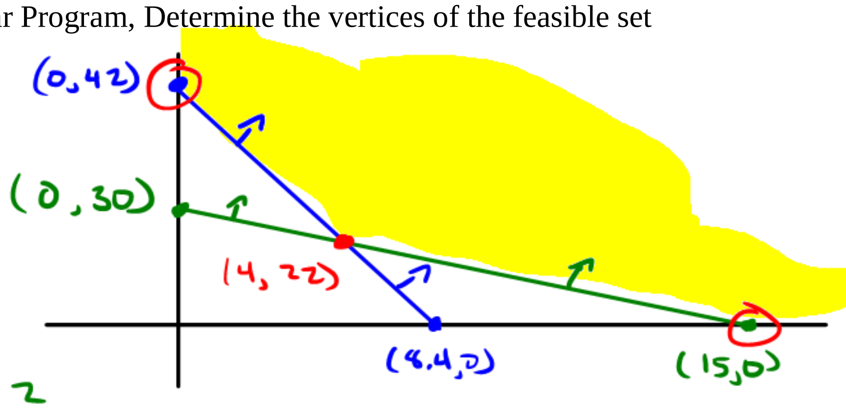
Example 2: Given the following Linear Program, Determine the vertices of the feasible set

Min $D_3 = 3x + y$

Subject to:
$10x + 2y \geq 84$ (1)
$8x + 4y \geq 120$ (2)
$x, y \geq 0$



line 1

x-int: $y=0$, $10x = 84$, $x = 8.4$

y-int: $x=0$, $2y = 84$, $y = 42$

$10x + 2y \geq 84$

$2y \geq -10x + 84$

$y \geq -5x + 42$

line 2

x-int: $y=0$, $8x = 120$, $x = 15$

y-int: $x=0$, $4y = 120$, $y = 30$

$8x + 4y \geq 120$

$4y \geq -8x + 120$

$y \geq -2x + 30$

$-5x + 42 = -2x + 30$

$-3x = -12$

$x = 4$

$y = 22$

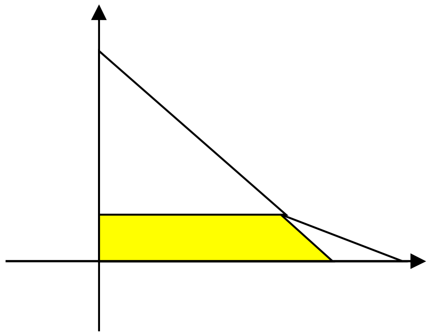
Corners of feasible set

$\boxed{(0,42),\ (4,22),\ (15,0)}$

Math 1313 Section 2.1

Popper 3

Question 1: The following graph represents a system of linear inequalities.



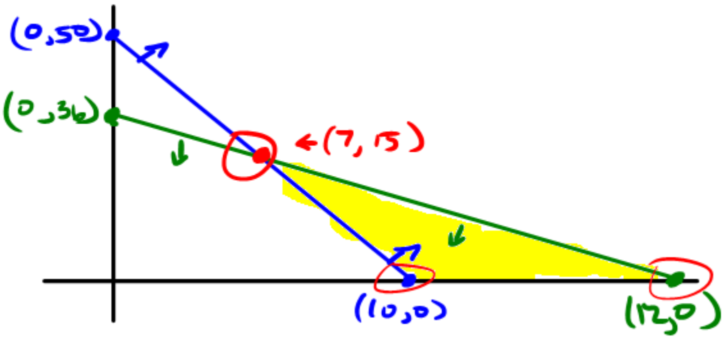
How many corner points does the following feasible set have?

a. 3
b. 5
c. 4
d. 6

Example 3: Given the following Linear Program, solve for the optimal solution.

Min $C(x, y) = 1200x + 100y$

Subject to:
$40x + 8y \geq 400$ 1
$3x + y \leq 36$ 2
$x, y \geq 0$



Line 1

x-int: $y = 0$, $40x = 400$, $x = 10$

y-int: $x = 0$, $8y = 400$, $y = 50$

$40x + 8y \geq 400$
$8y \geq -40x + 400$
$y \geq -5x + 50$
↑ Above

Line 2

x-int: $y = 0$, $3x = 36$, $x = 12$

y-int: $x = 0$, $y = 36$

$3x + y \leq 36$
$y \leq -3x + 36$
↑ Below

$-5x + 50 = -3x + 36$
$-2x = -14$
$x = 7$
$y = 15$

Pts	Min $C = 1200x + 100y$
(7, 15)	$1200(7) + 100(15) = 9{,}900$
(10, 0)	$1200(10) + 100(0) = 12{,}000$
(12, 0)	$1200(12) + 100(0) = 14{,}400$

Optimal Solution is 9,900 occurred (7, 15)

Example 4: Maximize the following Linear Programming Problem.

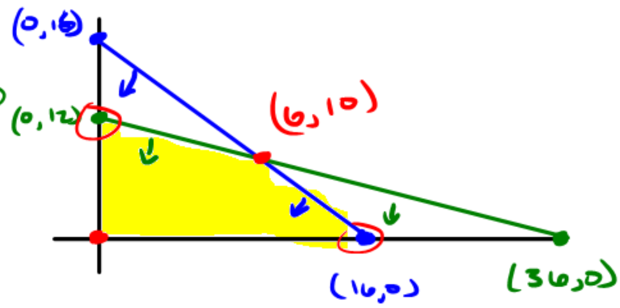
Maximize $C = 6x + 20y$

s.t. $x + y \leq 16$ ①

$x + 3y \leq 36$ ②

$x \geq 0$

$y \geq 0$



line 1

x-int	y-int
$y=0$	$x=0$
$x=16$	$y=16$

$x+y \leq 16$

$y \leq -x+16$

line 2

x-int	y-int
$y=0$	$x=0$
$x=36$	$3y=36$
	$y=12$

$x+3y \leq 36$

$3y \leq -x+36$

$y \leq -\frac{x}{3}+12$

$-x+16 = -\frac{x}{3}+12$

$-\frac{2}{3}x = -4$

$x = 6$

$y = 10$

	$\text{Max } C = 6x+20y$
$(0,12)$	$6(0)+20(12) = 240$
$(6,10)$	$6(6)+20(10) = 236$
$(16,0)$	$6(16)+20(0) = 96$
$(0,0)$	$6(0)+20(0) = 0$

Optimal Solution is 240 at (0,12)

Math 1313 Section 2.1

Example 5: Maximize the following Linear Programming Problem.

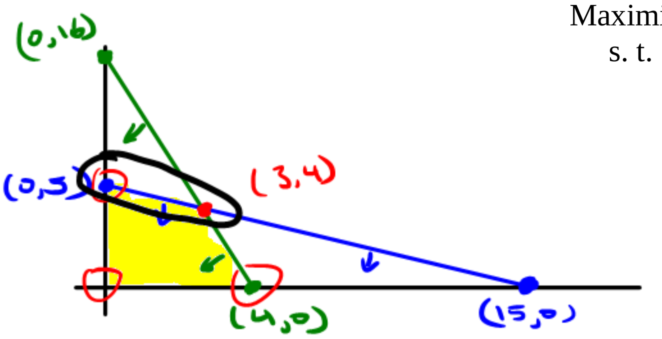
Maximize $C = 2x + 6y$

s. t. $x + 3y \le 15$ ①

$4x + y \le 16$

$x \ge 0$

$y \ge 0$



line 1

x-int: $y = 0$, $x = 15$

y-int: $x = 0$, $3y = 15$, $y = 5$

$x + 3y \le 15$

$3y \le -x + 15$

$y \le -\frac{x}{3} + 5$

line 2

x-int: $y = 0$, $4x = 16$, $x = 4$

y-int: $x = 0$, $y = 16$

$4x + y \le 16$

$y \le -4x + 16$

$-\frac{x}{3} + 5 = -4x + 16$

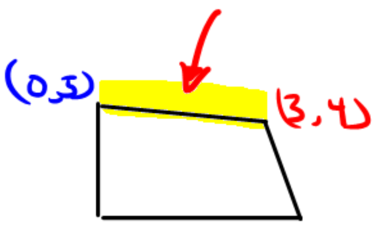
$\frac{11}{3}x = 11$

$x = 3$

$y = 4$

Pts	Max $C = 2x + 6y$	
(0,5)	$2(0) + 6(5) = 30$	Tie
(3,4)	$2(3) + 6(4) = 30$	
(4,0)	$2(4) + 6(0) = 8$	
(0,0)	0	

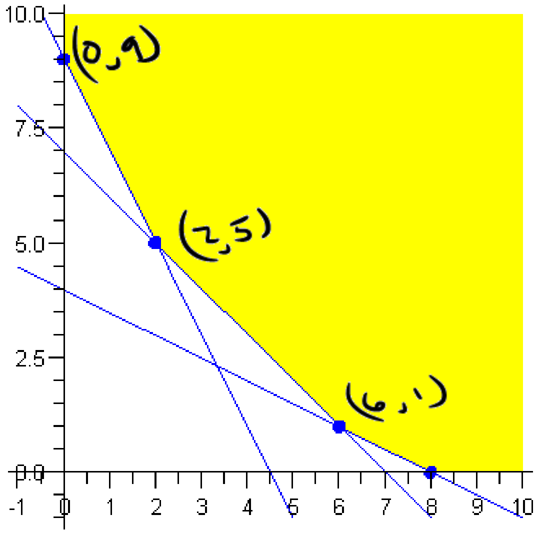
Infinitely Many solutions



Math 1313 Section 2.2

Popper 3

Question 3: Use the feasible set shown to determine which corner point minimizes the objective function C = 15x + 41y.



a. (0,0)
b. (2,5)
c. (6,1)
d. (8,0)

Section 2.2: Applications of Linear Programming

Example 1: The Soundex Company produces two models of mp3 players. Model A requires 15 min of work on assembly line I and 10 min of work on assembly line II. Model B requires 10 min of work on assembly line I and 12 min of work on assembly line II. At most, 23 hr of assembly time on line I and 22 hr of assembly time on line II are available per Soundex's work day. It is anticipated that Soundex will realize a profit of $12 on Model A and $10 on Model B. How many mp3 players of each model should be produced per day in order to maximize Soundex's profit?

a. Define your variables.

x – # of Model A   y – # of Model B

b. Construct and fill in a table.

1 hr = 60 min

	X	Y				
Line I	15 min	10 min	≤	23 hr	x 60	= 1380 min
Line II	10 min	12 min	≤	22 hr	x 60	= 1320 min
Profit	12	10				

c. State the Linear Programming Problem. Do Not Solve.

Max Profit $= 12x + 10y$

s.t.

$$15x + 10y \le 1380$$

$$10x + 12y \le 1320$$

$$x, y \ge 0$$

Example 2: A patient in a hospital is required to have at least 84 units of drug D1 and at least 120 units of drug D2 each day (assume that an overdose of either drug is harmless). Two substances, M and N, contain each of these drugs; however, in addition, both contain an undesirable drug D3 . Each gram of substance M contains 10 units of drug D1 , 8 units of drug D2 and 3 units of drug D3 . Each gram of substance N contains 2 units of drug D1 , 4 units of drug D2 and 1 unit of drug D3 . How many grams of substances M and N should be mixed to meet the minimum daily requirements and at the same time minimize the intake of drug D3 ?

a. Define your variables.

x – Grams of M   y – Grams of N

b. Construct and fill in a table.

	x	y		
D1	10	2	$\geq$	84
D2	8	4	$\geq$	120
D3	3	1		

c. State the Linear Programming Problem. Do Not Solve.

Min $D_3 = 3x + y$

st

$10x + 2y \geq 84$

$8x + 4y \geq 120$

$x, y \geq 0$

Example 3: The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs $1,200 to rent. Each van can transport 8 students, requires 1 chaperone, and cost $100 to rent. The officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. How many vehicles of each type should the officers rent in order to minimize the transportation costs? What are the minimal transportation costs?

a. Define your variables.

b. Construct and fill in a table.

c. State the Linear Programming Problem. Do Not Solve.

Math 1313 Section 2.2

Popper 3
Question 5
A certain academic department at a local university will conduct a research project. The department will need to hire graduate research assistants and professional researchers. Each graduate research assistant will need to work 26 hours per week on fieldwork and 14 hours per week at the university's research center. Each professional researcher will need to work 12 hours per week on fieldwork and 28 hours per week at the university's research center. The minimum number of hours needed per week for fieldwork is 158 and the minimum number of hours needed per week at the research center is 130. Each research assistant will be paid \$266 per week and each professional researcher will be paid \$452 per week. Let x denote the number of graduate research assistants hired and let y denote the number of professional researcher hired. The department wants to minimize cost.

Give a constraint of the problem.

a. $26x + 14y \geq 158$
b. $26x + 12y \geq 158$
c. $28x + 14y \leq 30$
d. $28x + 14y \geq 30$

Example 4: A 4-H member raises only geese and pigs. She wants to raise no more than 16 animals. It costs her \$ 500 to raise a goose and \$ 1500 to raise a pig. And she has \$ 18,000 available for the project. The 4-H member wishes to maximize her profit. Each goose produces \$ 600 in the profit and each pig \$ 2000 profit. How many of each animal should she raise to maximize her profit?

a. Define your variables.

b. Construct and fill in a table.

c. State the Linear Programming Problem and Solve