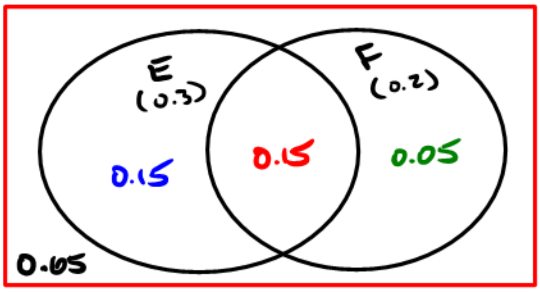
14MC

Math 1313 Sec. 6.5, 6.6 Ch 7

Review for test 4

1. Let E and F be two events of an experiment, P(E) = 0.30 and P(F) = 0.20 , and $P(E \cup F) = 0.35$. Find the following probabilities:

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$



a. $P(E \cap F) = P(E) + P(F) - P(E \cup F)$
$= 0.3 + 0.2 - 0.35$
$= 0.15$

b. $P(E^C \cap F) \rightarrow$ F only
$= 0.05$

Looking for

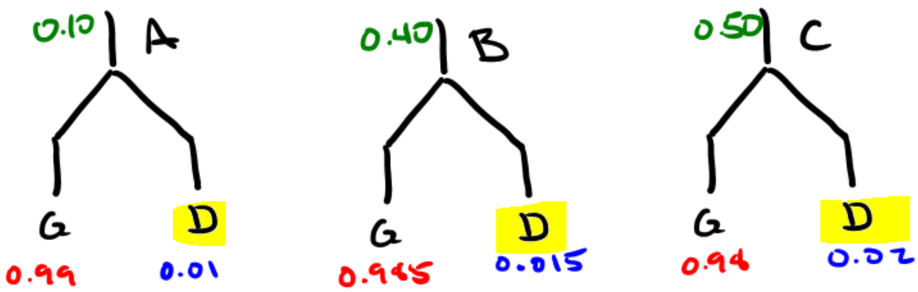
c. P(E | F) $= \dfrac{P(E \cap F)}{P(F)} = \dfrac{0.15}{0.2} = 0.75$

Known

d. $P(F|E^C) = \dfrac{P(F \cap E^C)}{P(E^C)} = \dfrac{0.05}{0.65 + 0.05} = \dfrac{0.05}{0.70} = 0.0714$

or $1 - P(E)$

2. Companies A, B. and C produce 10%, 40% and 50% respectively of the Model II computer. It has been found 1 % from A , 1½ % from B and 2% from C are found to be defective . Find the following probabilities:



a. Find the probability of a computer being defective.

$P(D) = 0.1(0.01) + 0.4(0.015) + 0.5(0.02) = \boxed{0.017}$

b. Find the probability of a computer being defective given it came from company C.

$P(D|C) = \dfrac{P(D \cap C)}{P(C)} = \dfrac{0.5(0.02)}{0.5} = \boxed{0.02}$

c. If it s found that a computer is defective, find the probability it came from company A.

$P(A|D) = \dfrac{P(A \cap D)}{P(D)} = \dfrac{0.10(0.01)}{0.1(0.01) + 0.4(0.015) + 0.5(0.02)} =$

$= \boxed{0.0588}$

Popper 27   1-5   E

Math 1313

3. The odds for rain tomorrow are 2:3. What is the probability of it not raining?

2:3 Rain

3:2 No Rain

$$P(\text{No Rain}) = \frac{3}{3+2} = \frac{3}{5} = \boxed{0.6}$$

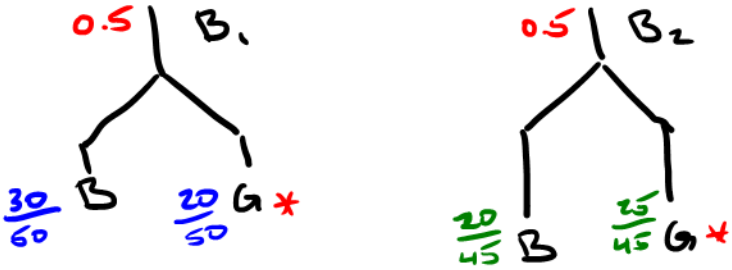
4. The probability of a horse winning is 0.4. What are the odds of the horse winning?

$P(\text{Win}) = 0.4$

$P(\text{Loss}) = 0.6$

$$\frac{P(\text{Win})}{P(\text{Loss})} = \frac{0.4}{0.6} = \frac{4}{6} = \frac{2}{3} \quad \boxed{2:3}$$

5. One of 2 bands is chosen at random with equally likely probability then a band member is chosen at random from the chosen band. Band one has 30 boys and 20 girls. Band two has 20 boys and 25 girls. Find the indicated probabilities:



a. What is the probability that band two was chosen, given a boy was chosen?

$$P(B_2 \mid \text{Boy}) = \frac{P(B_2 \cap \text{Boy})}{P(\text{Boy})} = \frac{0.5\left(\frac{20}{45}\right)}{0.5\left(\frac{30}{50}\right) + 0.5\left(\frac{20}{45}\right)} = \boxed{0.4255}$$

b. What is the probability that a girl was chosen?

$$P(\text{Girl}) = 0.5\left(\frac{20}{50}\right) + 0.5\left(\frac{25}{45}\right) = \boxed{0.4778}$$

c. What is the probability a boy was chosen given band one was chosen?

$$P(\text{Boy} \mid B_1) = \frac{P(\text{Boy} \cap B_1)}{P(B_1)} = \frac{0.5\,(30/50)}{0.5} = \frac{30}{50} = \boxed{0.6}$$

6. A 45 point quiz was given to a history class. The scores are listed below with the corresponding probability. Find the average for this class. μ

X	30	32	33	37	42
P(X = x)	0.15	0.225	0.175	0.3	0.1

$$E(X) = x_1 p_1 + x_2 p_2 + \dots x_n p_n$$

$$= 30(0.15) + 32(0.225) + \dots 42(0.1) = \boxed{32.775}$$

7. Given the following probability distribution with an expected value of 6.7. Find the standard deviation.

X	3	5	9	12
P(X = x)	0.4	0.2	0.1	0.3

$$\text{Var}(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \dots$$

$$= 0.4(3-6.7)^2 + 0.2(5-6.7)^2 + \dots = 15.01$$

$$\text{Stand. Dev.} = \sqrt{\text{Var}(X)} = \sqrt{15.01} = \boxed{3.8743}$$

Math 1313

$P(\mu - k\sigma < x < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

8. The heights of 4,000 women who participate in a recent survey have a mean of 64.5 inches and a standard deviation of 2.5. Use Chebychev's Inequality to estimate the probability that a woman chosen at random height will be between 60.5 and 68.5. $P(60.5 < x < 68.5) \geq 1 - \frac{1}{k^2}$

$\mu = 64.5$
$\sigma = 2.5$

$\mu - k\sigma = $ Small
$\mu + k\sigma = $ Big
Solve for K.

$64.5 + 2.5K = 68.5$

$2.5k = 4$
$k = 1.6 = \frac{8}{5}$

$1 - \frac{1}{(1.6)^2}$

$= \boxed{0.60975}$

9. Consider the binomial experiment. The probability that cell phone is defective is 0.11. If a sample of 6 cell phones is selected at random.

$n = 6 \quad p = 0.11 \quad q = 0.89$

$P(X = x) = C(n, x)\, p^x q^{n-x}$

a. What is the probability at least two are defective? $P(x \geq 2) = x = 2, 3, 4, 5, 6$ Too Much Work

$= 1 - P(X < 2) = 1 - [P(x=0) + P(x=1)]$

$= 1 - [C(6,0)(0.11)^0(0.89)^6 + C(6,1)(0.11)^1(0.89)^5] \quad = \boxed{0.1345}$

b. What is the mean or expected value?

$E(X) = np = 6(0.11) \quad = \boxed{0.66}$

c. What is the Variance and standard deviation?

$var(X) = npq$
$= 6(0.11)(0.89)$
$= \boxed{0.5874}$

$\sigma = \sqrt{npq}$
$= \sqrt{6(0.11)(0.89)}$
$= \boxed{0.7664}$

10. Consider the following binomial experiment. The probability that a person will get a cold this winter is 0.55. A sample of 10 people were chosen random. $n = 10 \quad p = 0.55 \quad q = 0.45$

$P(x = x) = C(n, x)\, p^x q^{n-x}$

a. Find the probability that is at least 2 people will get a cold.

$P(x \geq 2) = 1 - P(x < 2) = 1 - [P(x=0) + P(x=1)]$

$1 - [C(10,0)(0.55)^0(0.45)^{10} + C(10,1)(0.55)^1(0.45)^9] \quad = \boxed{0.9955}$

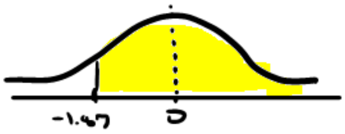
b. Find the probability that is exactly five people will get a cold

$P(X=5) = C(10,5)(0.55)^5(0.45)^5 \quad = \boxed{0.2340}$

11. Let Z be a standard normal random variable. Find the following probabilities:

a. P(Z< - 1.47) ← Look it up

$= \boxed{0.0708}$



Math 1313

b. P(Z > - 1.87) $P(Z < 1.87)$ or $1 - P(Z < -1.87)$

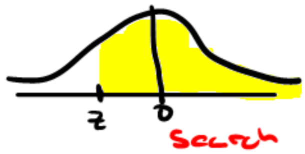
$= 0.9693$

c. P(1.1< Z < 2.13) $= P(Z < 2.13) - P(Z < 1.1)$

$0.9834 - 0.8643$

$= \boxed{0.1191}$

d. P(Z < z) = .8264 search for this $\boxed{z = 0.94}$



e. P(Z >z) = .8665 $P(Z < -z) = 0.8665$ $1 - P(Z > z) = 0.1335$ ← search for this

$\boxed{z = -1.11}$

f. P(-z < Z < z) = .8690

$= \frac{1}{2}(1 + P(-z < Z < z))$

$= \frac{1}{2}(1 + 0.8690)$

$= \frac{1}{2}(1.8690)$

$= 0.9345$ ← search for this

$\boxed{z = 1.51}$

z table →

12. The heights of award winning tomatoes plants were normally distributive with a mean of 10 inches and a standard deviation of 2 inches. Find the probability that a plant selected at random measures between 8 and 12.

$\mu = 10$
$\sigma = 2$

$P(8 < X < 12) = P\left(\frac{8-10}{2} < Z < \frac{12-10}{2}\right)$

$= P(-1.00 < Z < 1.00)$

$P(Z < 1.00) - P(Z < -1.00)$

$0.8413 - 0.1587 = \boxed{0.6826}$

z tables

13. The test scores on the last exam for the students in Finite were normally distributive with a mean 72 and a standard deviation of 10. What is the probability that a student scored below a 60?

$\mu = 72$
$\sigma = 10$

$P(X < 60)$

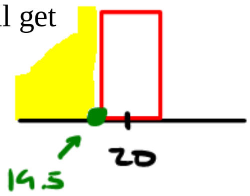
$P\left(Z < \frac{60 - \mu}{\sigma}\right) = P\left(Z < \frac{60-72}{10}\right)$

$= P(Z < -1.20)$

$= 0.1151$

Math 1313 "0.5 increment"

14. Use the normal distribution to approximate the binomial distribution. A flu vaccine has a probability of 20% of not preventing a person who is inoculated from getting the flu. A county health office inoculated 134 people. Find the probability that fewer than 20 will get the flu.



$P(X<20) \approx P(Y<19.5)$

$p=0.2$ $q=0.8$

$n=134$

$\mu=np=134(0.2)$

$=26.8$

$\sigma=\sqrt{npq}=\sqrt{134(0.2)(.8)}$

$=4.6303$

At least 4 Decimals

$P\left(Z<\frac{19.5-26.8}{4.6303}\right)$

$=P(Z<-1.58)$ * Two Decimal

$=\boxed{0.0571}$