110 min

30 MC

Math 1313 Final Exam Review Ch 1 - 7

Example 1: Find the equation of the line containing points (1,2) and (2,3).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-2}{2-1} = \frac{1}{1} = 1$

$y = mx + b$

$2 = 1(1) + b$

$2 = 1 + b$

$1 = b$

$\boxed{y = x + 1}$

Example 2: The Ace Company installed a new machine in one of its factories at a cost of \$20,000. The machine is depreciated linearly over 10 years with a scrap value of \$2,000. Find the value of the machine after 5 years.

$m = \frac{\text{Scrap} - \text{Initial}}{\text{time}}$

$= \frac{2{,}000 - 20{,}000}{10}$

$= -1{,}800$

$V(t) = mt + \text{Initial}$

$V(t) = -1800t + 20{,}000$

$V(5) = -1800(5) + 20{,}000 = \boxed{\$11{,}000}$

Example 3: The AC Florist Company got a new delivery van at a cost of \$28,000. The van is depreciated linearly over 5 years and has no scrap value. Find the value of the machine after 2 years.

$m = \frac{\text{Scrap} - \text{Initial}}{\text{time}}$

$= \frac{0 - 28{,}000}{5}$

$= -5{,}600$

$V(t) = mt + \text{Initial}$

$V(t) = -5{,}600t + 28{,}000$

$V(2) = -5{,}600(2) + 28000 = \boxed{\$16{,}800}$

Example 4: 4. A manufacturer has a monthly fixed cost of \$1200 and a production cost of \$2.50 for each unit produced. The product sells for \$10 per unit.

a. What is the cost function?

$C(x) = cx + F = \boxed{2.5x + 1200}$

b. What is the revenue function?

$R(x) = Sx = \boxed{10x}$

c. What is the profit function?

$P(x) = R(x) - C(x) = (s - c)x - F = (10 - 2.5)x - 1200$

$= \boxed{7.5x - 1200}$

d. What is the break- even point?

$R(x) = C(x)$

$10x = 2.5x + 1200$

$7.5x = 1200$

$x = 160$

Break-Even Quantity

$R(x) = 10x$

$R(160) = 10(160)$

$= 1600$

Break-Even Revenue

$\boxed{(160, 1600) \text{ Break-Even Point}}$

Example 5: Solve using Gauss-Jordon.

$$x - 5y + z = 5$$
$$-y + z = 2$$
$$3x + 2y + z = 11$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 3 & 2 & 1 & 11 \end{array}\right) \xrightarrow{-3R_1+R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 0 & 17 & -2 & -4 \end{array}\right) \xrightarrow{-R_2 \to R_2}$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 17 & -2 & -4 \end{array}\right) \begin{array}{l} 5R_2+R_1 \to R_1 \\ -17R_2+R_3 \to R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 15 & 30 \end{array}\right) \xrightarrow{\frac{1}{15}R_3 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} 4R_3+R_1 \to R_1 \\ R_3+R_2 \to R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$x = 3$
$y = 0$
$z = 2$

Example 6: Given the following matrices are in row reduced form. State the solution, if it exists to the system of equations.

$$\begin{array}{cccc} x & y & z & \\ \hline \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 6 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$x = -2$
$y = 1$
$z = 6$

$x = 2$
$y + 2z = 0$; $y = -2z$
$0 = 0$ $z = ??$
Infinitely Many

$x = -1$
$y = 3$
$0 = 4$ X
No Solution

Example 7: Solve for a, b, c and d.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -2 & 4 \end{bmatrix}$$

$a - 2 = 4$
$a = 6$

$b - (-1) = 3$
$b = 2$

$c - (-5) = -2$
$c = -7$

$d - 6 = 4$
$d = 10$

Example 8: Find the transpose of the following matrices.

2×3

$$A = \begin{bmatrix} -2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

3×3

$$B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$A^T = 3 \times 2$

$$= \begin{pmatrix} -2 & 1 \\ 3 & 0 \\ 2 & 4 \end{pmatrix}$$

$B^T = 3 \times 3$

$$= \begin{pmatrix} 2 & 0 & 1 \\ 2 & -1 & 2 \\ 1 & 3 & 4 \end{pmatrix}$$

Example 9: Find the product, if possible.

$$\begin{bmatrix}1 & -1 & 0 & 1\\ 2 & 1 & 2 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 0\\ -1 & 2\end{bmatrix}$$

$4 \neq 3$

$(2 \times 4)\ (3 \times 2)$

Not Defined ; Not Possible

Example 10: Find the product, if possible

$$\begin{bmatrix}2 & 1\\ 1 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -1\end{bmatrix}$$

$(3 \times 2)(2 \times 3)$

$= 3 \times 3$

$R_1 \cdot C_1$: $2+3$ | $R_1 \cdot C_2$: $-2+0$ | $R_1 \cdot C_3$: $4-1$

$R_2 \cdot C_1$: $1+0$ | $R_2 \cdot C_2$: $-1+0$ | $R_2 \cdot C_3$: $2+0$

$R_3 \cdot C_1$: $-1+6$ | $R_3 \cdot C_2$: $1+0$ | $R_3 \cdot C_3$: $-2-2$

$$= \boxed{\begin{pmatrix}5 & -2 & 3\\ 1 & -1 & 2\\ 5 & 1 & -4\end{pmatrix}}$$

Example 11: Find the inverse of the following matrix

$$\begin{bmatrix}5 & 3\\ -4 & 6\end{bmatrix}$$

Determinant = $ad-bc$

$= 5(6) - 3(-4)$

$= 30+12$

$= 42 \neq 0$

Inverse Exist

$$A^{-1} = \frac{1}{D}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} = \frac{1}{42}\begin{bmatrix}6 & -3\\ 4 & 5\end{bmatrix}$$

$$= \begin{pmatrix}\frac{6}{42} & \frac{-3}{42}\\ \frac{4}{42} & \frac{5}{42}\end{pmatrix} = \boxed{\begin{pmatrix}\frac{1}{7} & \frac{-1}{14}\\ \frac{2}{21} & \frac{5}{42}\end{pmatrix}}$$

Example 12: A manufacturer of stereo speakers, makes two kinds of speakers, an economy model which sells for \$50 and a deluxe model which sells for \$200. The deluxe model uses 1 woofer and 2 tweeters. The economy uses 1 woofer and 1 tweeter. The manufacturer currently has 20 woofers and 45 tweeters in inventory. Set-up the problem to maximize income from the sale, use x for economy and y for deluxe.

	X	Y	
Woofer	1	1	≤ 20
Tweeter	1	2	≤ 45
Profit	50	200	

Max $P = 50x + 200y$

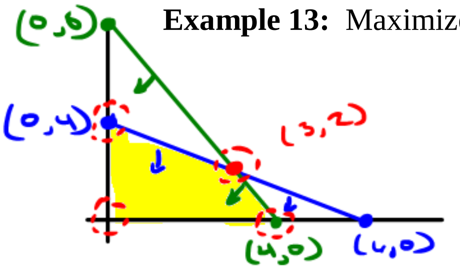
s.t.

$x + y \leq 20$

$x + 2y \leq 45$

$x, y \geq 0$

Example 13: Maximize the following Linear Programming Problem.



$$\begin{aligned} Max\ P &= 3x + 2y \\ st\colon\ 2x + 3y &\le 12 \quad ① \\ 2x + y &\le 8 \quad ② \\ x, y &\ge 0 \end{aligned}$$

x-int	y-int
(6,0)	(0,4)
(4,0)	(0,8)

$2x+3y \le 12$

$3y \le -2x+12$

$y \le -\frac{2}{3}x+4$

$2x+y \le 8$

$y \le -2x+8$

$-\frac{2}{3}x+4 = -2x+8$

$\frac{4}{3}x = 4$

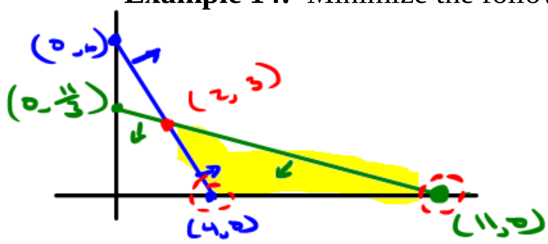
$x = 3$

$y = 2$

Pts	Max $P = 3x+2y$
(0,4)	$3(0)+2(4) = 8$
(3,2)	$3(3)+2(2) = 13$
(4,0)	$3(4)+2(0) = 12$
(0,0)	$3(0)+2(0) = 0$

Optimal Value is 13 occurs at (3,2)

Example 14: Minimize the following Linear Programming Problem.



$$\begin{aligned} Min\ C &= x + y \\ st\colon\ 3x + 2y &\ge 12 \quad ① \\ x + 3y &\le 11 \quad ② \\ x, y &\ge 0 \end{aligned}$$

x-int	y-int
(4,0)	(0,6)
(11,0)	$(0, \frac{11}{3})$

$3x+2y \ge 12$

$2y \ge -3x+12$

$y \ge -\frac{3}{2}x+6$

$x+3y \le 11$

$3y \le -x+11$

$y \le -\frac{1}{3}x+\frac{11}{3}$

$-\frac{3}{2}x+6 = -\frac{1}{3}x+\frac{11}{3}$

$-\frac{7}{6}x = -\frac{7}{3}$

$x = 2$ $\quad y = 3$

Pts	Min $C = x+y$
(2,3)	$2+3 = 5$
(4,0)	$4+0 = 4$
(11,0)	$11+0 = 11$

Optimal Value is 4 at (4,0)

ch 4

Example 15: Find the accumulated amount at the end of 6 months on a \$2000 bank deposit paying simple interest at a rate of 3% per year.

$F = P(1+rt)$

$= 2000\left(1 + 0.03\left(\frac{6}{12}\right)\right)$

$= \$2{,}030$

Example 16: Dave invested a sum of money 3 years ago in a savings account that has since paid interest at the rate of 4.5% per year compounded monthly. His investment is now worth $5,721.24. How much did he originally invest?

P.V.

One time Deposit

P.V. w/ compound Interest

$P = F(1+i)^{-n}$

$= 5,721.24\,(1 + 0.045/12)^{-36}$

$= \boxed{\$5,000}$

Annuity

Example 17: Mike pays $300 per month for 4 years for a car, making ~~no down payment.~~ If the loan borrowed costs 7% per year compounded monthly, what was the original cost of the car? How much interest will be paid?

P.V.A

$P = E\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

P.V.

$P = 300\left(1-(1+0.07/12)^{-48}\right)/(0.07/12) = 12,528.06$

$300 × 48 (4 yrs) = 14,400

Interest = 14400 − 12528.06 = $\boxed{1471.94}$

+ DP

$\boxed{12,528.06}$

Example 18: Steve bought a car for $30,000 . He put down 10% and financed the balance. His bank charged him 5% compounded monthly for 5 years. What is the monthly payment?

E ??

Amort or ~~S.F.~~

$E = \dfrac{Pi}{(1-(1+i)^{-n})}$

P = 30000 − 10%

= 27000

$27000 * 0.05/12 / (1 - (1+0.05/12)^{-60})$

$= \boxed{504.52}$

Example 19: George decided to deposit $4,000 to pay for a cruise he plans to take in 2 years. His bank pays 3.5% annual interest compounded semiannually. How much will he have in his account at the end of two years?

F.V.

One time Deposit

Fv w/ compound Interest

$F = P(1+i)^n$

$= 4000(1 + 0.035/2)^4$

$= \boxed{4287.44}$

Example 20: Sandy decided to save some money for her daughter's college education. She decided to save $500 per quarter. Her credit union pays 4.5% annual interest compounded quarterly. How much money will she have available when her daughter starts college in 10 years?

Annuity

FVA: $F = E\left[\dfrac{(1+i)^n - 1}{i}\right] = 500\left((1+0.045/4)^{40} - 1\right)/(0.045/4) = \boxed{25,083.42}$

ch 5

Example 21: Let U={1,2,3,4,5,6,7,8,9,10},A = {1,3,5,7,9}, B={2,4,6,8,10}, C={1,2,4}

a. $B \cap C^c$

In Common

{2,4,6,8,10} ∩ {3,5,6,7,8,9,10}

= $\boxed{\{6,8,10\}}$

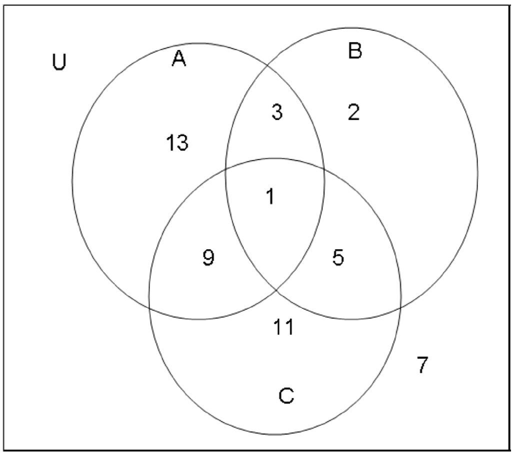
b. $A \cup C^c$

{1,3,5,7,9} ∪ {3,5,6,7,8,9,10}

Merge

$\boxed{\{1,3,5,6,7,8,9,10\}}$

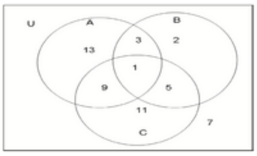
Example 22: Given the Venn Diagram.



a. $n[B \cup (A^c \cap C^c)]$

$B \cup (A \cup C)^c = 3+2+1+5+7$

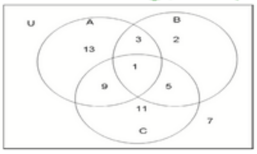
$= \boxed{18}$



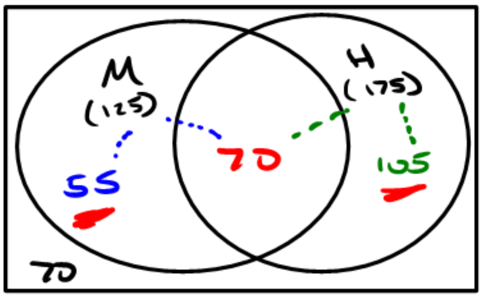
b. $n[B \cap (A^C \cap C^c)]$

$B \cap (A \cup C)^c = \boxed{2}$

In Common



Example 23: In a group of 300 hundred students, 125 are currently taking a math class and 175 are taking a history class and 70 are taking both classes. How many students in this group are taking a math class or a history class but not both?.



$55 + 105$

$= \boxed{160}$

Example 24: Suppose a person planning a banquet cannot decide how to seat 6 honored guests at the head table. In how many arrangements can they seated in the 6 chairs on one side of the table?

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ $\quad 6! = 720$

P (# of Chairs, # of ppl) = P(6,6)

Example 25: In how many ways can a president, vice president, secretary, and treasurer be selected from an organization of 20 members?

$n = 20$
$r = 4$

Titles $\rightarrow P(20,4) = \boxed{116{,}280}$

Example 26: You are going to make a serial number which can have no repeats and contains 3 digits and two letters. A zero cannot be the first digit. How many serials numbers are possible?

$\underline{9}$ 1-9 $\quad \underline{9}$ Not 1st $\quad \underline{8}$ Not 1st or 2nd $\quad \underline{26} \quad \underline{25}$ Not the 1st $\quad = \boxed{421{,}200}$

Example 27: A car dealer is offering special pricing on a truck. It has four models, six exterior colors, 3 interior colors, four choices of seat coverings and 3 stereo systems. If you can only choose one in each category, how many different trucks could be constructed?

$\underset{\text{Model}}{4}\ \underset{\text{Ext}}{6}\ \underset{\text{Int}}{3}\ \underset{\text{Seat}}{4}\ \underset{\text{System}}{3} = \boxed{864}$

Example 28: Find the number of ways in which 8 members of the space shuttle crew can be selected from 20 available astronauts.

Nothing Specific $\rightarrow C(20,8) = \boxed{125,970}$

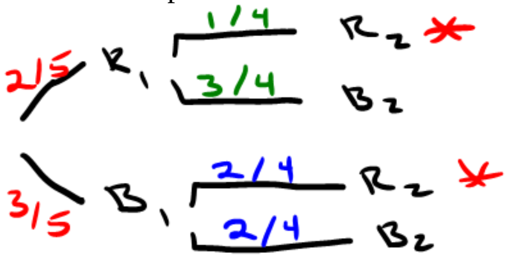
b. The command structure on a space flight is determined by the order in which astronauts are selected for the flight. How many different command structures are possible if 8 astronauts are selected from 20 that are available?

Order Matters $\rightarrow P(20,8) = \boxed{5,079,110,400}$

c. If 14 men and 6 women are available for a space shuttle flight, in how many crews are possible that have 5 men and 3 women?

$C(14,5)\,C(6,3) = \boxed{40,040}$

Example 29: A box contains 2 red marbles and 3 black marbles. Two marbles are drawn in succession without replacement. Find the following:



a. Find the probability the second marble is red?

$P(\text{2nd is Red}) = \frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right) = 0.4$

b. Find the probability that both marbles are the same color?

$P(RR \text{ or } BB) = \frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right) = 0.4$

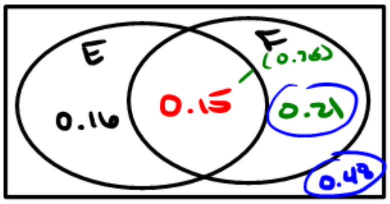
c. Find the probability that the second marble is black given the first marble is red?

$P(B_2 \mid R_1) = \frac{P(B_2 \cap R_1)}{P(R_1)} = \frac{\cancel{2/5}(3/4)}{\cancel{2/5}} = \frac{3}{4} = 0.75$

d. Find the probability that first marble red given that the second marble is red?

$P(R_1 \mid R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{\frac{2}{5}\left(\frac{1}{4}\right)}{\frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right)} = \boxed{0.25}$

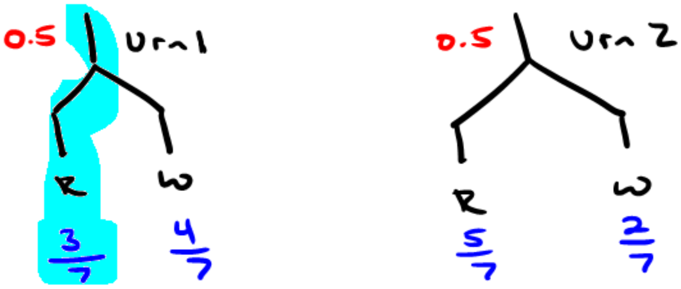
Example 30: Let E and F be events of a sample space S. Let $P(E^C) = 0.69$, $P(F) = 0.36$ and $P(E \cap F) = 0.15$. Find $P(E \cup F)$. ← Two Circles



$= 0.16 + 0.15 + 0.21$

$= \boxed{0.52}$

Example 31: Urn I contains 3 red and 4 white marbles and Urn II contains 5 red and 2 white marbles. Each Urn has an equally likely probability of being chosen. Find the following probabilities if a marble is chosen:



a. What is the probability that Urn I is selected and a red marble?

$0.5\left(\frac{3}{7}\right) = \boxed{0.2143}$

b. What is the probability that a red marble is chosen?

$0.5\left(\frac{3}{7}\right) + 0.5\left(\frac{5}{7}\right) = \boxed{0.5714}$

c. What is the probability that Urn I is selected given that a red marble has been selected?

$P(U_1 | R) = \frac{P(U_1 \cap R)}{P(R)} = \frac{0.5(3/7)}{0.5\left(\frac{3}{7}\right) + 0.5\left(\frac{5}{7}\right)} = \boxed{0.375}$

d. What is the probability that a white marble is chosen given that Urn II was selected?

$P(W | U_2) = \frac{2}{7} = \boxed{0.2857}$

Example 32. A sample of 6 fuses is drawn from a lot containing 10 fuses and 2 defective fuses. Find the probability that the number of defective fuses is: $n(S) = C(12, 6) = 924$

10 G 2D

a. Exactly 1? $C(2,1)\,C(10,5) = 504$ $\quad \frac{504}{924} = 0.5455$

b. No defective fuses?

$\frac{2D}{0}$ $\frac{10G}{6}$ $\quad C(2,0)\,C(10,6)$ $= 210$ $\quad P(E) = \frac{210}{924} = 0.2273$

c. At least 1 defective fuses?

↳ Use Complement (0 Def)

$P(E) = 1 - P(E^C) = 1 - \frac{210}{924} = 0.7727$

Example 33: The probability distribution for a random variable X is given below. Calculate the expected value. $E(x) = \mu = p_1 x_1 + p_2 x_2 + x_n p_n$

X	14	16	18	20
P(X=x)	0.34	0.31	0.26	0.09

$= 0.34(14) + 0.31(16) + 0.26(18) + 0.09(20) = 16.2$

Example 34: Consider the following Binomial experiment. The probability that a new employee at a manufacturing plant is still employed after one year is 0.9. Seven people have recently been hired by the company. $n = 7$ $p = 0.9$ $q = 0.1$

$P(X=x) = C(n,x)\, p^x q^{n-x}$

a. What is the probability that exactly 4 of these new employees will still be employed after one year? $P(X=4) = C(7,4)(0.9)^4(0.1)^3 = \boxed{0.02296}$

b. What is the probability that at least 6 of the new employee's will still be employed after one year? $P(X \geq 6) = P(X=6) + P(X=7)$

$= C(7,6)(0.9)^6(0.1)^1 + C(7,7)(0.9)^7(0.1)^0 = \boxed{0.85031}$

c. Calculate the mean of new employees that will still be employed after one year?

$E(X) = \mu = np = 7(0.9) = \boxed{6.3}$

d. Calculate the standard deviation.

$\sigma = \sqrt{npq} = \sqrt{7(0.9)(0.1)} = \boxed{0.7937}$

Example 35: Z is a standard normal random variable.

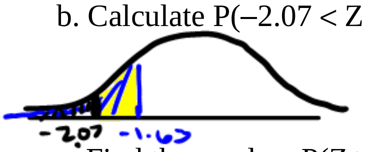
↙ Use Symmetry

a. Calculate P(Z > 0.19) . $= P(Z < -0.19) = \boxed{0.4247}$

b. Calculate P(–2.07 < Z < –1.63) . $P(Z < -1.63) - P(Z < -2.07)$

$0.0516 - 0.0192$

$= \boxed{0.0324}$



c. Find the z value, P(Z > z) = .9115

$P(Z < -z) = 0.9115$

$-z = 1.35$

$\boxed{z = -1.35}$

d. Find the value of z, P(–z < Z < z) = .8444

$P(Z < z) = \frac{1}{2}(1 + P(-z < Z < z))$

$= \frac{1}{2}(1 + 0.8444)$

$= \frac{1}{2}(1.8444)$

$= 0.9222$ ↙ Search for it

$\boxed{z = 1.42}$

Example 36: Suppose X is a normal random variable with $\mu = 380$ and $\sigma = 20$. Find the value of:

a. $P(X < 405) = P\left(Z < \frac{405 - 380}{20}\right)$

$= P(Z < 1.25) = \boxed{0.8944}$

b. $P(X > 330) = P\left(Z > \frac{330 - 380}{20}\right)$

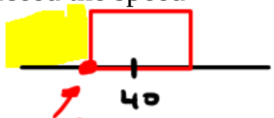
$P(Z > -2.50)$ Use Symmetry

$P(Z < 2.50) = \boxed{0.9938}$

0.5 increment

Example 37: Use the normal distribution to approximate the following binomial distribution. Consider the random sample of 100 drivers on interstate 10 in Texas, where 29% of the drivers exceed the 70 mph speed limit. Find the probability that fewer than 40 drivers exceed the speed limit.

$n = 100$ $p = 0.29$ $q = 0.71$



$P(X < 40) \approx P(Y < 39.5)$

$\mu = np$
$= 100(0.29) = 29$

$\sigma = \sqrt{npq} =$
$= \sqrt{100(0.29)(0.71)}$
$= 4.5376$
At least 4 Decimals

$P\left(Z < \frac{39.5 - 29}{4.5376}\right)$

$P(Z < 2.3139)$ * Round to 2 Decimals

$P(Z < 2.31)$

$= \boxed{0.9896}$