$\mu = 400$ $\sigma = 50$

Section 7.6: Applications

Example 1: According to the data released by the Chamber of Commerce of a certain city, the weekly wages of factory workers are normally distributed with a mean of $400 and a standard deviation of $50. Find the probability that a factory worker selected at random from this city makes a weekly wage of

a. more than $460?

$P(X > 460) = P\left(Z > \frac{460-400}{50}\right) = P(Z > 1.20) = 0.1151$

$1 - P(Z < 1.20)$

$P(Z < -1.20)$

b. between $350 and $450?

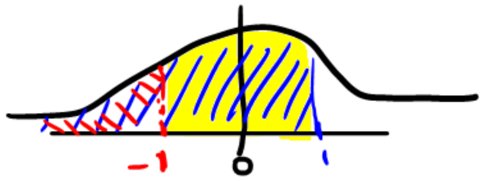
$P(350 < X < 450) \Rightarrow P\left(\frac{350-400}{50} < Z < \frac{450-400}{50}\right)$

$= P(-1.00 < Z \leq 1.00)$

$= P(Z < 1.00) - P(Z < -1.00)$

$= 0.8413 - 0.1587$

$= 0.6826$



"0.5 step increment"

Theorem
Suppose we are given a binomial distribution associated with a binomial experiment involving *n* trials, each with probability of success *p* and probability of failure *q*. Then if *n* is large and *p* is not close to 0 or 1, the binomial distribution may be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$.

Example 2: Consider the following binomial experiment. A marksman's chance of hitting a target with each of his shots is 60%. If he fires 30 shots, what is the probability of his hitting the target between 15 and 20 times, inclusive?

$P(15 \leq X \leq 20) \approx P(14.5 < Y < 20.5)$

$n = 30$ $p = 0.6$ $q = 0.4$

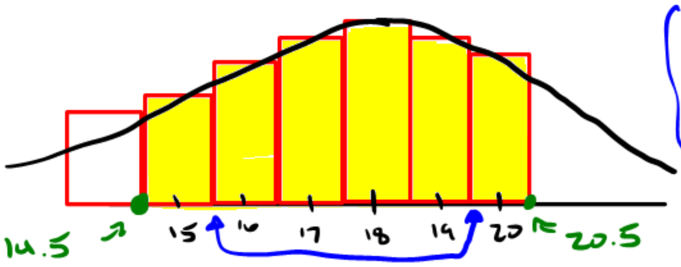
$\mu = np = 30(0.6) = 18$

$\sigma = \sqrt{npq}$

$= \sqrt{30(0.6)(0.4)}$

≈ 2.6833

At least 4 Decimals



"$P(15 < X < 20)$

$\approx (15.5 < Y < 19.5)$

Extra Info

$P\left(\frac{14.5-18}{2.6833} < Z < \frac{20.5-18}{2.6833}\right)$

$= P(-1.30 < Z < 0.93)$ ← Round 2 Decimals

$= P(Z < 0.93) - P(Z < -1.30)$

$= 0.8238 - 0.0968$

$= \boxed{0.7270}$

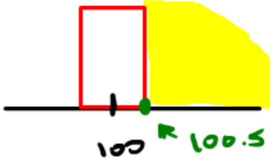
Math 1313 Section 7.6 "0.5 increment"

Example 3: Use the normal distribution to approximate the binomial distribution. A basketball player has a 75% chance of making a free throw. They will take 120 attempts. What is the probability of them making:

$p = 0.75$ $q = 0.25$ $n = 120$

$\mu = np = 90$ $\sigma = \sqrt{npq} = 4.7434$

a. more than 100 free throws?



$P(X > 100) \approx P(Y > 100.5)$

$= P\left(Z > \frac{100.5 - 90}{4.7434}\right)$

$= P(Z > 2.21)$ Two Decimals places

Symmetry: $P(Z < -2.21)$

Complement: $1 - P(Z < 2.21)$

$\boxed{= 0.0136}$

b. Fewer than 85 free throws?

$P(X < 85) \approx P(Y < 84.5)$

$= P\left(Z < \frac{84.5 - 90}{4.7434}\right)$

$= P(Z < -1.16)$

$= \boxed{0.1230}$

"0.5 increment"

Example 4: Use the normal distribution to approximate the binomial distribution. A die is rolled 84 times. What is the probability that the number 4 occurs more than 13 times?

$n = 84$

"Success is getting 4"

$p = \frac{1}{6}$ $q = \frac{5}{6}$

$\mu = np = 14$

$\sigma = \sqrt{npq} = 3.4157$

$P(X > 13) \approx P(Y > 13.5)$

$= P\left(Z > \frac{13.5 - 14}{3.4157}\right)$

$= P(Z > -0.15)$

Symmetry: $P(Z < 0.15)$

Complement: $1 - P(Z < -0.15)$

$= \boxed{0.5596}$