Section 7.5: The Normal Distribution

A random variable that may take on infinitely many values is called a **continuous random variable**.

The probability distribution associated with this type of random variable is called a **continuous probability distribution.**

A continuous probability distribution is defined by a function f called the **probability density function**. The function has domain equal to those values the continuous random variable assumes.

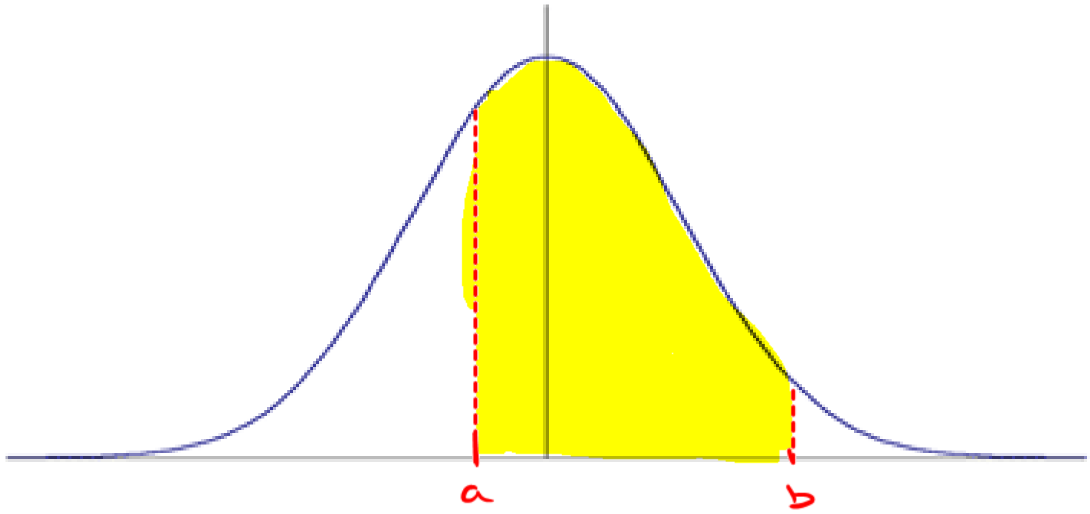
The **probability density function** has the following properties:

1. f(x) >0 for all values of x.

2. The area between the curve and the x axis is 1.

The probability that the random variable X associated with a given probability density function assumes a value in an interval $a < x < b$ is given by the **area of the region between the graph of *f* and the *x*-axis from *x* = *a* to *x* = *b*.**

Here is a picture:



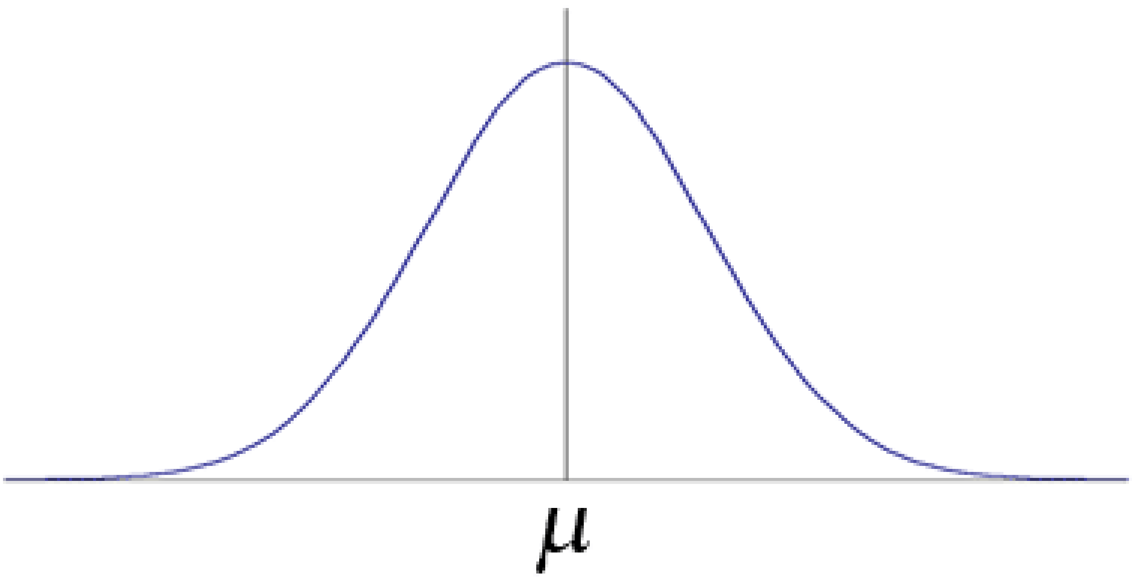
This value is $P(a < X < b)$

Note: $P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b) = P(a < X < b)$, since the area under one point is 0.

Normal Distributions.

For these types of distributions:
1. The graph is a bell-shaped curve.
2. μ and σ each have the same meaning (mean and standard deviation)
3. μ determines the location of the center of the curve.
4. σ determines the sharpness or flatness of the curve.

Also, the normal curve has the following characteristics:
1. The curve has peak at $x = \mu$.
2. The curve is symmetric with respect to the vertical line $x = \mu$.
3. The curve always lies above the x-axis but approaches the x-axis as x extends indefinitely in either direction.
4. The area under the curve is 1.
5. For any normal curve, 68.27% of the area under the curve lies within 1 standard deviation of the mean (i.e. between $\mu - \sigma$ and $\mu + \sigma$), 95.45% of the area lies within 2 standard deviations of the mean, and 99.73% of the area lies within 3 standard deviations of the mean.



Since any normal curve can be transformed into any other normal curve we will study, from here on, the **Standard Normal Curve**.

The **Standard Normal Curve** has $\mu = 0$ and $\sigma = 1$.

The corresponding distribution and random variable are called **the Standard Normal Distribution** and the **Standard Normal Random Variable**, respectively.
The Standard Normal Variable will commonly be denoted Z.

$P(Z < \text{Numbers})$

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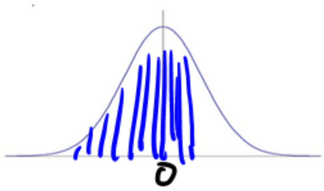
The **area of the region under the standard normal curve to the left** of some value z, i.e. $P(Z < z)$ or $P(Z \leq z)$, is calculated for us in **Table I**.

Example 1: Let Z be the standard normal variable find the values of:

a. $P(Z < -1.91)$

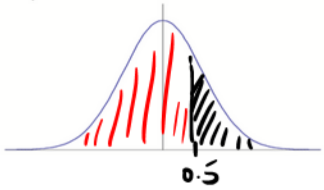
$= 0.0281$

b. $P(Z < 0.44)$



$= 0.6700$

c. $P(Z > 0.50)$

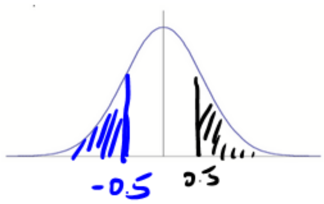


complement

$= 1 - P(Z < 0.50)$

$= 1 - 0.6915$

$= 0.3085$

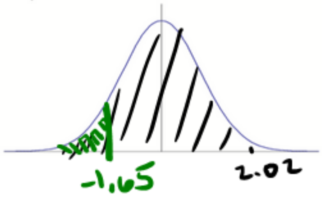


Symmetry

$= P(Z < -0.50)$

$= 0.3085$

d. $P(-1.65 < Z < 2.02)$

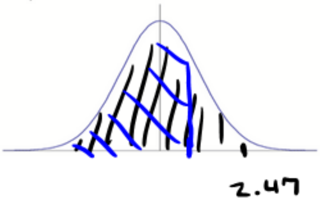


$P(Z < 2.02) - P(Z < -1.65)$

$= 0.9783 - 0.0495$

$= 0.9288$

e. $P(1 < Z < 2.47)$



$P(Z < 2.47) - P(Z < 1)$

$0.9932 - 0.8413$

$= 0.1519$

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Popper 3: $P(Z > 0.22)$

a. 0.6578
b. 0.2879
c. 0.4129
d. 0.5488
e. 0.7602

Example 2: Let Z be the standard normal variable. Find the value of z if z satisfies:

a. $P(Z < z) = 0.9495$ ← Search for this

$z = 1.64$

b. $P(Z > z) = 0.9115$

$P(Z < z) = 1 - 0.9115$

$= 0.0885$

$z = -1.35$

Symmetry $P(Z < \boxed{-z}) = 0.9115$

$\boxed{-z} = 1.35$

$z = -1.35$

c. $P(Z < -z) = 0.6950$

$-z = 0.51$

$z = -0.51$

d. $P(-z < Z < z) = 0.7888$

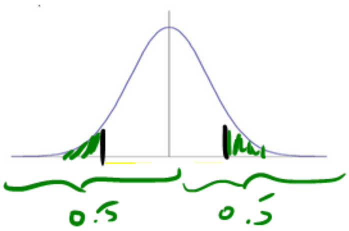
$P(Z < \text{Number}) = \frac{1}{2}(1 + P(-z < Z < z))$

$= \frac{1}{2}(1 + 0.7888)$

$= \frac{1}{2}(1.7888)$

$= 0.8944$ ← Search for this

$z = 1.25$



e. P(-z < Z < z) = 0.8444

$P(Z < z) = \frac{1}{2}(1 + P(-z < Z < z))$

$= \frac{1}{2}(1 + 0.8444)$

$= \frac{1}{2}(1.8444)$

$= 0.9222$

$z = 1.42$

$0.5 - \frac{0.6444}{2}$

$0.5 - 0.4222$

$= 0.0778$

Popper 4: Let Z be the standard normal variable. Find the value of z if z satisfies $P(Z > z) = 0.7054$.

a. 0.46
b. -0.46
c. -0.54
d. 1.24
e. None of the above

When given a normal distribution in which $\mu \neq 0$ and $\sigma \neq 1$, we can transform the normal curve to the standard normal curve by doing whichever of the following applies.

$$P(X < b) = P\left(Z < \frac{b-\mu}{\sigma}\right)$$

$$P(X > a) = P\left(Z > \frac{a-\mu}{\sigma}\right)$$

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

Example 3: Suppose X is a normal variable with $\mu = 80$ and $\sigma = 10$. Find:

a. $P(X < 100)$ $\Rightarrow P\left(Z < \frac{100 - 80}{10}\right)$

$= P\left(Z < \frac{20}{10}\right)$

$= P(Z < 2.00) = 0.9772$

b. $P(X > 65)$ $\Rightarrow P\left(Z > \frac{65-80}{10}\right)$

$= P\left(Z > \frac{-15}{10}\right)$

$= P(Z > -1.50) = P(Z < 1.50) = 0.9332$

c. $P(70 < X < 95)$ $\Rightarrow P\left(\frac{70-80}{10} < Z < \frac{95-80}{10}\right)$

$= P(-1.00 < Z < 1.50)$

$= P(Z < 1.50) - P(Z < -1.00)$

$= 0.9332 - 0.1587$

$= 0.7745$

Popper 5: Suppose X is a normal random variable with mean of 560 and standard deviation 40. Find the probability of more than 510.

$P(X > 510)$

a. 0.1248
b. 0.8944
c. 0.6514
d. 0.3486
e. None of the above