Chebychev's Inequality

Let X be a random variable with expected value μ and standard deviation σ. Then, the probability that a randomly chosen outcome of the experiment lies between $\mu - k\sigma$ and $\mu + k\sigma$ is at least $1 - \frac{1}{k^2}$; that is, $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}$

1st step
Solve for
k

2nd
Find the
Approximation

Example 3: A probability distribution has a mean 20 and a standard deviation of 3. Use Chebychev's Inequality to estimate the probability that an outcome of the experiment lies between 12 and 28.

$P(12 \le x \le 28)$

$\mu = 20$
$\sigma = 3$

$\mu - k\sigma = \text{Small}$
$20 - k(3) = 12$
$-3k = -8$
$k = \frac{8}{3}$

$\mu + k\sigma = \text{Big}$
$20 + k(3) = 28$
$3k = 8$
$k = 8/3$

$P(12 < x < 28) \ge 1 - \frac{1}{k^2}$

$\ge 1 - \frac{1}{(8/3)^2} = \boxed{0.859375}$

Example 4: A light bulb has an expected life of 200 hours and a standard deviation of 2 hours. Use Chebychev's Inequality to estimate the probability that one of these light bulbs will last between 190 and 210 hours?

$\mu = 200$
$\sigma = 2$

$P(190 \le x \le 210)$

Solve for k first.

$\mu + k\sigma = \text{Big \#}$
$200 + 2k = 210$
$2k = 10$
$k = 5$

$P(190 \le x \le 210) \ge 1 - \frac{1}{k^2}$

$\ge 1 - \frac{1}{(5)^2} = 0.96$

Popper 3: A light bulb at an art museum has an expected life of 300 hours and a standard deviation of 12 hours. Use Chebychev's Inequality to estimate the probability that one of these light bulbs will last between 280 and 320 hours of use.

a. 0.6400
b. 0.6000
c. 0.4000
d. 0.3600

Section 7.4: The Binomial Distribution

A binomial experiment has the following properties:

1. Number of trials is fixed.
2. There are 2 outcomes of the experiment. Success, probability denoted by p, and failure, probability denoted by q. Note $p + q = 1$
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Experiments with two outcomes are called **Bernoulli trials** or **Binomial trials**.

Finding the Probability of an Event of a Binomial Experiment:

In a binomial experiment in which the probability of success in any trial is p, the probability of exactly x successes in n independent trials is given by

p is raised to success

$$P(X = x) = C(n, x)p^x q^{n-x}$$

q is raised to failures

X is called a **binomial random variable** and its probability distribution is called a **binomial probability distribution**. Example 1 in section 7.4 derives this formula.

Example 1: Consider the following binomial experiment. A fair die is cast four times. Compute the probability of obtaining exactly one 6 in the four throws.

$P(X = \# \text{ of success}) = C(n, \text{success})\, p^x q^{n-x}$

$n = 4$

"Success" = Rolling a 6

$p = \frac{1}{6}$

Failure

$q = \frac{5}{6}$

1 success

$P(X = 1) = C(4, 1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{4-1}$

$= C(4, 1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3$

$= \boxed{0.3858}$

$n=5$

Example 2: Let the random variable *X* denote the number of girls in a five-child family. If the probability of a female birth is 0.6, construct the binomial distribution associated with this experiment.

$p=0.6$
$q=0.4$

0, 1, 2, 3, 4, 5

X	$P(X=x)=C(n,x)\,p^x\,q^{n-x}$	
0	$C(5,0)(0.6)^0(0.4)^5$	$=0.01024$
1	$C(5,1)(0.6)^1(0.4)^4$	$=0.0768$
2	$C(5,2)(0.6)^2(0.4)^3$	$=0.2304$
3	$C(5,3)(0.6)^3(0.4)^2$	$=0.3456$
4	$C(5,4)(0.6)^4(0.4)^1$	$=0.2592$
5	$C(5,5)(0.6)^5(0.4)^0$	$=0.07776$

Should Add to 1

Example 3: Consider the following binomial experiment. If the probability that a marriage will end in divorce within 20 years after its start is 0.6, what is the probability that out of 6 couples just married, in the next 20 years

$p=0.6$ $\quad q=0.4$ $\quad n=6$

a. all will be divorced?

$$P(X=6)=C(6,6)(0.6)^6(0.4)^0=\boxed{0.046656}$$

b. None will be divorced?

$$P(X=0)=C(6,0)(0.6)^0(0.4)^6=\boxed{0.004096}$$

c. Exactly two couples will be divorced?

$$P(X=2)=C(6,2)(0.6)^2(0.4)^4=\boxed{0.13824}$$

d. At least two couples will be divorced? Using the Complement

$$P(X\geq 2)=1-P(X<2)$$
$$=1-[P(X=0)+P(X=1)]$$
$$=1-\Big[\underbrace{C(6,0)(0.6)^0(0.4)^6}_{P(X=0)}+\underbrace{C(6,1)(0.6)^1(0.4)^5}_{P(X=1)}\Big]$$

$$=\boxed{0.95904}$$

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Popper 4: Consider the following binomial experiment. A company owns 4 copiers. The probability that on a given day any one copier will break down is 23/50. What is the probability that 2 copiers will break down on a given day?

a. 0.3702
b. 0.4576
c. 0.5805
d. 0.6102

Mean, Variance and Standard Deviation of a Random Variable

If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p, and probability of failure q, then the mean E(X), variance and standard deviation of X are given by applying the following formulas:

$$\mu = E(X) = np$$

$$Var(X) = npq$$

$$\sigma = \sqrt{Var(X)} = \sqrt{npq}$$

Example 4: The probability of a person contracting influenza on exposure is 62%. In the binomial experiment for a family of 12 that has been exposed, what is the: $p = 0.62$ $q = 0.38$

$n = 12$

a. mean? $E(X) = np = 12(0.62)$

$= 7.44$

b. standard deviation? $\sigma = \sqrt{npq} = \sqrt{12(0.62)(0.38)}$

$= 1.6814$

c. variance? $Var(X) = npq = 12(0.62)(0.38)$

$= 2.8272$

d. probability that at most 10 contract influenza?

$P(X \le 10) = P(X=0) + P(X=1) + \ldots + P(X=10)$

Complement

$= 1 - P(X > 10) = 1 - [P(X=11) + P(X=12)]$

$1 - [C(12,11)(0.62)^{11}(0.38)^{1} + C(12,12)(0.62)^{12}(0.38)^{0}]$

$= 0.9730$

Popper 5: Consider the following binomial experiment. The probability that a fuse produced by a certain company will be defective is 11/50. If 500 fuses are produced each day, how many can we expect to find each day that are defective?

E(x)

a. 107
b. 110
c. 112
d. 113