Section 7.2: Expected Value and Odds

The **average** (mean) of n numbers, $x_1, x_2, x_3, \ldots, x_n$ is $\bar{x}$.

Formula: $\bar{\mathbf{x}} = \dfrac{\mathbf{x_1 + x_2 + \ldots + x_n}}{\mathbf{n}}$

Expected Value of a Random Variable *X*

Let X denote a random variable that assumes the values $x_1, x_2, x_3, \ldots, x_n$ with associated probabilities p_1 $p_2, \ldots, p_n$, respectively. The **expected value** of X, $E(X)$, is given by

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n = \mu$$

Mu

Example 1: In this Finite Math class Test 1 is worth 8%, each of the 2- 4 tests are worth 14%, the quiz average is worth 12%, the homework average is worth 10%, the popper average is worth 10%, and the final exam is worth 18%. If your grades are as follows:

Test 1 – 82, Test 2 – 86, Test 3 – 68, Test 4 – 90, Quiz Average – 100, Homework Average – 91, Popper Average – 100, and Final Exam – 95.

What is your class average (expected value)?

$E(X) = x_1p_1 + x_2p_2 + \ldots + x_np_n$

$= 82(0.08) + 86(0.14) + 68(0.14) + 90(0.14) + 100(0.12) + 91(0.10) + 100(0.10) + 95(0.18)$

$= \boxed{88.92}$ B+

Example 2: A box contains 15 quarters, 7 dimes, 5 nickels, and 8 pennies. A coin is drawn at random from the box. What is the mean of the value of the draw?

$E(X)$

35 Total coins

$E(X) = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4$

$= 0.25\left(\frac{15}{35}\right) + 0.10\left(\frac{7}{35}\right) + 0.05\left(\frac{5}{35}\right) + 0.01\left(\frac{8}{35}\right)$

$= \boxed{0.1364}$

Example 3: An investor is interested in purchasing an apartment building containing six apartments. The current owner provides the following probability distribution indicating the probability that the given number of apartments will be rented during a given month.

Number of Rented Apt	0	1	2	3	4	5	6
Probability	0.02	0.08	0.10	0.12	0.15	0.25	0.28

a. Find the number of apartments the investor could expect to be rented during a given month?

$$E(x) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$
$$= 0(0.02) + 1(0.08) + 2(0.10) + \ldots + 6(0.28)$$
$$= 4.17$$

b. If the monthly rent for each apartment is $799, how much could the investor expect to collect in rent for the whole building during a given month?

# of Apartment × Rent

4.17×799

$= \$3331.83$

Popper 3: A box contains 17 nickels, 11 dimes and 19 pennies. You pick a coin at random from the box, what is the average value of the draw?

a. 0.0214
b. 0.0455
c. 0.1567
d. 0.7133

Odds in Favor of and Odds Against

If P(E) is the probability of an even E occurring, then

1. The odds in favor of E occurring are

$$\frac{P(E)}{P(E^c)}$$

2. The odds against E occurring are

$$\frac{P(E^c)}{P(E)}$$

Note: Odds are expressed as ratios of whole numbers.

"a to b"

a : b

Probability of an Event (Given the Odds)

If the odds in favor of an event E occurring are *a* to *b*, then the probability of E occurring is

$$P(E) = \frac{a}{a+b} = \text{decimal}$$

Example 4: The probability that the race horse Galloping George will win a race is 0.65.

$P(\text{Win}) = 0.65 \quad P(\text{Loss}) = 0.35$

a. What are the odds in favor of George winning?

$$\frac{P(\text{win})}{P(\text{loss})} = \frac{0.65}{0.35} = \frac{65}{35} = \frac{13}{7} \quad \boxed{13:7}$$

odds of a win

b. What are the odds against George winning?

$$\frac{P(\text{Loss})}{P(\text{win})} = \frac{0.35}{0.65} = \frac{35}{65} = \frac{7}{13} \quad \boxed{7:13}$$

odd of a Loss

c. What is the probability that George will lose?

using odd of a Loss

$$P(\text{Loss}) = \frac{7}{7+13} = \frac{7}{20} = \boxed{0.35}$$

Example 5: The odds against Laura winning a certain raffle are 99:1. What is the probability that Laura will not win the raffle?

$$P(\text{Not Winning}) = \frac{99}{99+1} = 0.99$$

Example 6: The odds that it will rain on Thursday are 3 to 5. What is the probability that it will rain?

$$P(\text{rain}) = \frac{3}{3+5} = \frac{3}{8} = 0.375$$

Example 7: The probability that I will not finish my paper this week is 85%. What are the odds I will finish my paper?

$$P(\text{No Paper}) = 0.85$$

$$P(\text{Paper}) = 0.15$$

$$\frac{P(\text{Paper})}{P(\text{No Paper})} = \frac{0.15}{0.85} = \frac{15}{85}$$

$$= \frac{3}{17} \qquad \boxed{3:17}$$

Popper 4: The odds that a child entering a convenience store with her parents will not get a package of cupcakes are 1 to 11. What is the probability she will get a package of cupcakes?

a. 0.0833
b. 0.0909
c. 0.1236
d. 0.9167

Section 7.3: Variance and Standard Deviation

The **Variance** of a random variable X is the measure of degree of dispersion, or spread, of a probability distribution about its mean (i.e. how much on average each of the values of X deviates from the mean).

Note: A probability distribution with a small (large) spread about its mean will have a small (large) variance.

Variance of a Random Variable X
Suppose a random variable has the probability distribution

x	x_1	x_2	$\dots$	x_n
$P(X=x)$	p_1	p_2	$\dots$	p_n

and expected value $E(X) = \mu$. Then the variance of the random variable X is

$$Var(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \dots + p_n(x_n-\mu)^2$$

Note: We square each since some may be negative.

Standard Deviation measures the same thing as the variance. The standard deviation of a Random Variable X is

$$\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + \dots + p_n(x_n-\mu)^2}$$

Example 1: Compute the mean, variance and standard deviation of the random variable X with probability distribution as follows:

X	$P(X=x)$
-3	0.4
2	0.3
5	0.3

$E(X) = x_1p_1 + x_2p_2 + x_3p_3$

$= -3(0.4) + 2(0.3) + 5(0.3) = \boxed{0.9 = \mu}$

$var(X) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + p_3(x_3-\mu)^2$

$= 0.4(-3-0.9)^2 + 0.3(2-0.9)^2 + 0.3(5-0.9)^2$

$= \boxed{11.49}$

Stand. Dev. $= \sigma = \sqrt{var(X)} = \sqrt{11.49}$

$= \boxed{3.3857}$

Example 2: An investor is considering two business ventures. The anticipated returns (in thousands of dollars) of each venture are described by the following probability distributions:

Venture 1

Earnings	Probability
-5	0.2
30	0.6
60	0.2

Venture 2

Earning	Probability
1.5	0.15
50	0.75
100	0.10

a. Compute the mean and variance for each venture.

$E(X_1) = x_1 p_1 + x_2 p_2 + x_3 p_3 = -5(0.2) + 30(0.6) + 60(0.2) = 29 = \mu_1$

$Var(X_1) = p_1(x_1-\mu)^2 + p_2(x_2-\mu)^2 + p_3(x_3-\mu)^2$

$= 0.2(-5-29)^2 + 0.6(30-29)^2 + 0.2(60-29)^2 = 424$

$E(X_2) = 1.5(0.15) + 50(0.75) + 100(0.10) = 47.725$

$Var(X) = 0.15(1.5-47.725)^2 + 0.75(50-47.725)^2 +$

$0.10(100-47.725)^2 = 597.6619$

b. Which investment would provide the investor with the higher expected return (the greater mean)?

$\mu_1 = 29$
$\mu_2 = 47.725$

Venture 2

c. Which investment would the element of risk be less (that is, which probability distribution has the smaller variance)?

$Var(X_1) = 424$
$Var(X_2) = 597.6619$

Venture 1

Popper 5: The probability distribution of a random variable X is given below. Given the mean $\mu = 4$

X	1	3	5	7
$P(X = x)$	0.15	0.35	0.35	0.15

Find the standard deviation.

a. 3.400
b. 10.830
c. 1.844
d. 3.291