

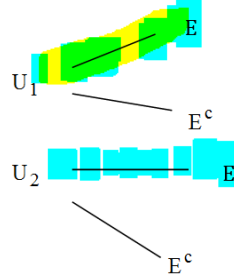
Math 1313 Section 6.6
Section 6.6 – Bayes’ Theorem

In section 6.5, we were interested in finding the probability of the second event on a tree diagram if we know that the first event had occurred. In this section, we reverse the roles. If we know the second event has occurred, what is the probability of a given outcome of the first event?

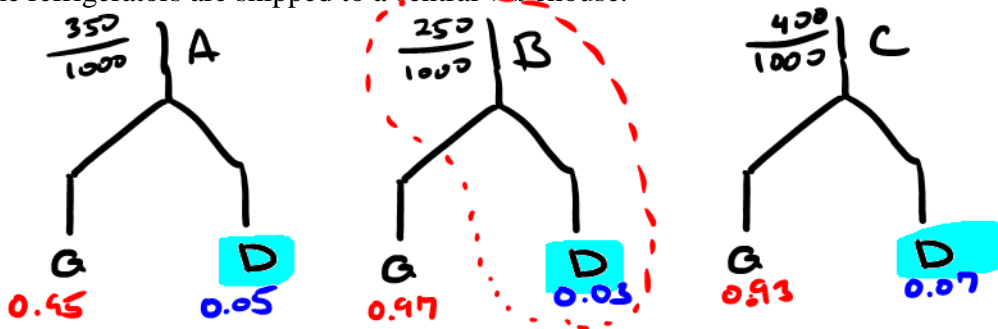
U_1 and U_2 are mutually exclusive events. We would like to know: $P(U_1 | E) = \frac{\text{Product of the branch probabilities leading to } E \text{ through } U_1}{\text{Sum of all branch products leading to } E}$

$$P(U_1 | E) = \frac{P(U_1 \cap E)}{P(U_1 \cap E) + P(U_2 \cap E)}$$

$P(E)$



Example 1: A company produces 1,000 refrigerators a week at three plants. Plant A produces 350 refrigerators a week, plant B produces 250 refrigerators a week, and plant C produces 400 refrigerators a week. Production records indicate that 5% of the refrigerators produced at plant A will be defective, 3% of those produced at plant B will be defective, and 7% of those produced at plant C will be defective. All the refrigerators are shipped to a central warehouse.



- a. What is the probability that a refrigerator chosen at random from the warehouse will be defective?

$$P(D) = \frac{350}{1000} (0.05) + \frac{250}{1000} (0.03) + \frac{400}{1000} (0.07)$$

$$= 0.053$$

- b. If a refrigerator at the warehouse is found to be defective, what is the probability that it was produced at plant B?

$$P(B | D) = \frac{P(B \cap D)}{P(D)}$$

$$= \frac{\frac{250}{1000} (0.03)}{0.053} = 0.1415$$

Part A

Math 1313 Section 6.6

Example 2: In a random sample of 1,000 people it was found that 7% have a liver ailment. Of those who have a liver ailment, 40% are heavy drinkers, 50% are moderate drinkers and 10% are nondrinkers. Of those who do not have a liver ailment, 10% are heavy drinkers, 70% are moderate drinkers and 20% are nondrinkers. If a person is chosen at random and it is found that he or she is a heavy drinker, what is the probability of that person having a liver ailment?

$$\begin{array}{l}
 0.07 \text{ LA} \\
 \hline
 0.40 \text{ HD}^* \\
 0.50 \text{ MD} \\
 0.10 \text{ ND}
 \end{array}$$

$$\begin{array}{l}
 0.93 \text{ LA}^c \\
 \hline
 0.10 \text{ HD}^* \\
 0.70 \text{ MD} \\
 0.20 \text{ ND}
 \end{array}$$

$$\begin{aligned}
 & P(\text{LA} | \text{HD}) \\
 &= \frac{P(\text{LA} \cap \text{HD})}{P(\text{HD})} \\
 &= \frac{0.07(0.40)}{0.07(0.4) + 0.93(0.10)} \\
 &= \boxed{0.2314}
 \end{aligned}$$

Popper 3: 11 yellow balls and 14 red balls are placed in an urn. Two balls are then drawn in succession without replacement. What is the probability that the first ball drawn is a red ball if the second ball drawn is yellow?

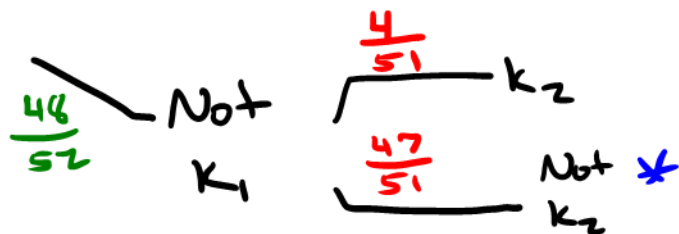
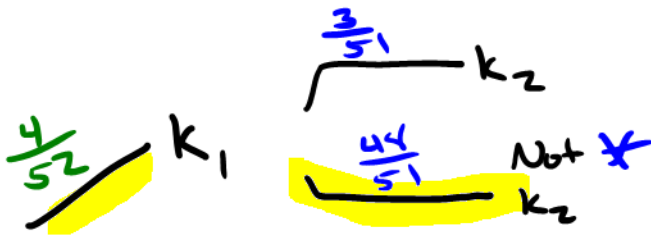
$$P(\text{1st is Red} | \text{2nd Yellow})$$

- 0.4400
- 0.5833
- 0.6000
- 0.9167

Math 1313 Section 6.6

Example 3: Suppose that from a well-shuffled deck of 52 playing cards two cards are drawn in succession, without replacement. What is the probability that the first card was a king, given that the second card was not a king?

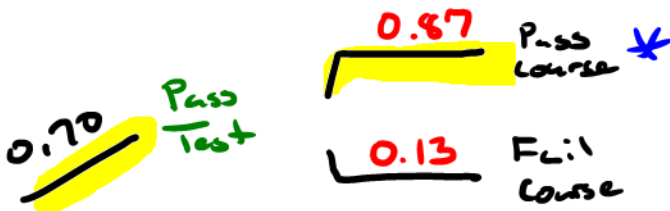
$$P(K_1 | \text{Not } K_2)$$



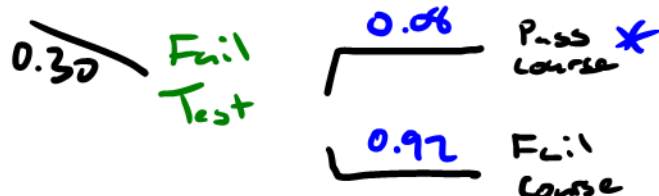
$$\begin{aligned} & \frac{P(K_1 \cap \text{Not } K_2)}{P(\text{Not } K_2)} \\ &= \frac{\frac{4}{52} \cdot \frac{48}{51}}{\frac{4}{52} \cdot \frac{48}{51} + \frac{48}{52} \cdot \frac{47}{51}} \\ &= 0.0784 \end{aligned}$$

Example 4: A placement test is given by a certain high school to predict student success in a particular math course. On average, 70% of students who take the test pass it, and 87% of those who pass the test also pass the course, whereas 8% of those who fail the test pass the course. If a student passed the course, what is the probability that he or she passed the test?

$$P(\text{Pass Test} | \text{Pass Course})$$



$$\frac{P(P T \cap P C)}{P(P C)}$$



$$\begin{aligned} & \frac{(0.70)(0.87)}{0.70(0.87) + 0.30(0.08)} \\ &= \boxed{0.9621} \end{aligned}$$

Math 1313 Section 6.6

Popper 4: Urn A contains 7 yellow balls and 3 red balls. Urn B contains 10 yellow balls and 7 red balls. Urn C contains 12 yellow balls and 4 red balls. An urn is picked randomly (assume that each urn is equally likely to be chosen), and then a ball is picked from the selected urn. What is the probability the chosen ball came from urn B knowing that it was a yellow ball?

- a. 0.0703
- b. 0.2886
- c. 0.3434
- d. 0.3680

$P(B | \text{Yellow})$



Section 7.1: Random Variables

A rule that assigns a number to each outcome of an experiment is called a random variable.

We can construct the probability distribution associated with a random variable

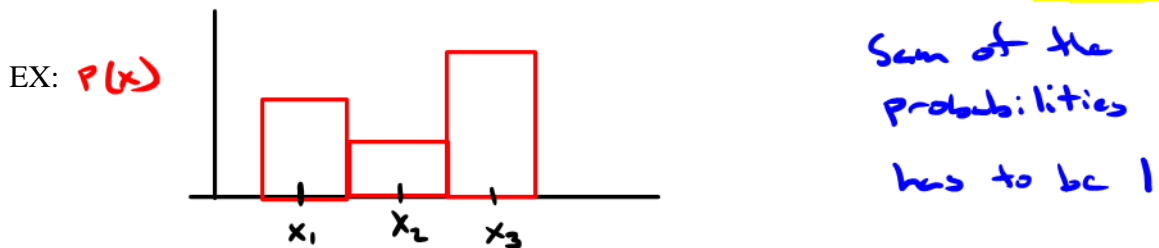
If $x_1, x_2, x_3, \dots, x_n$ are values assumed by the random variable X with associated probabilities $P(X=x_1), P(X=x_2), \dots, P(X=x_n)$, respectively, then the probability distribution of X may be expressed in the following way.

x	$P(X=x)$
x_1	$P(X=x_1)=p_1$
x_2	$P(X=x_2)=p_2$
\vdots	\vdots
\vdots	\vdots
x_n	$P(X=x_n)=p_n$

random variables { } Probabilities

We can also graphically represent the probability distribution of a R.V.

A bar graph which represents the probability distribution of a random variable is called a histogram.



Example 1: The probability distribution of the random variable X is shown in the accompanying table:

x	-2	-1	0	1	2	3
$P(X=x)$	0.01	0.11	0.20	0.32	0.21	0.15

Find:

a. $P(X = -2) = 0.01$

b. $P(-1 \leq X < 1) = P(X = -1) + P(X = 0) = 0.11 + 0.20 = 0.31$

c. $P(X > 0) = P(X = 1) + P(X = 2) + P(X = 3) = 0.32 + 0.21 + 0.15 = 0.68$

Example 2: A survey was conducted by the Public Housing Authority in a certain community among 920 families to determine the distribution of families by size.

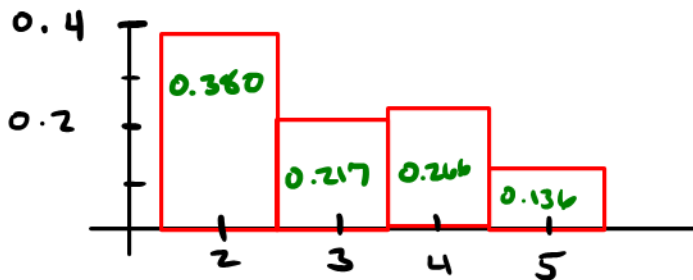
The results follow:

Family Size	2	3	4	5	
Frequency of Occurrence	350	200	245	125	← Add the occurrences

a. Let X denote the number of persons in a randomly chosen family. Find the probability distribution for this experiment.

X	2	3	4	5
$P(X)$	$\frac{350}{920}$ 0.380	$\frac{200}{920}$ 0.217	$\frac{245}{920}$ 0.266	$\frac{125}{920}$ 0.136

b. Draw the histogram corresponding to the probability distribution in part a.



c. What is the $P(3 < X \leq 5)$? = $P(X=4) + P(X=5)$
 = $\boxed{0.402}$

d. What is the $P(X > 2)$? = $P(X=3) + P(X=4) + P(X=5)$ or $1 - P(X=2)$
 = $\boxed{0.620}$

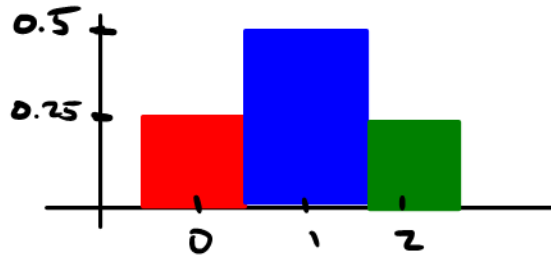
e. What is the $P(2 \leq X \leq 5)$? = $\underbrace{P(X=2) + P(X=3) + P(X=4) + P(X=5)}_{\text{All possibilities}}$
 = 1

Example 3: A coin is tossed twice. Let the random variable X denote the number of tails that occur in the two tosses. Find the probability distribution for X and then draw the histogram corresponding to the probability distribution of X .

$X = \# \text{ of Tails}$

$HH \leftarrow 0 \text{ Tails}$
 $HT \left\{ \begin{array}{l} 1 \text{ Tail} \\ TH \end{array} \right.$
 $TT \leftarrow 2 \text{ Tails}$

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$



Popper 5: Using the information from the above problem, Find the $P(X \leq 1)$

- a. 0.250
- b. 0.500
- c. 0.750
- d. 1.000