50 min

14 MC

Ch 4, 5, 6.1 – 6.4

Test 3 Review

For questions 1 –6, state the type of problem and calculate the answer. → Annuity

1. Parents of a college student wish to set up an account that will pay $350 per month to the student for four years. How much should they deposit now at 9% annual interest, compounded monthly?

↳ P.V.

P.V.A.

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$E = 350$

$m = 12$

$t = 4$

$n = mt = 48$

$r = 0.09$

$i = \frac{r}{m} = \frac{0.09}{12}$

$= \boxed{14,064.67}$

Annuity

2. Your friend's payments on his new car are $524.37 per month. He received a $3000 trade-in on his old car, and received a financing package that was 8.9% annual interest, compounded monthly for five years. What was the total purchase price of the car?

↳ P.V.

$$E\left(1-(1+i)^{-n}\right)/(i)$$

$$524.37\left(1-(1+0.089/12)^{-60}\right)/(0.089/12)$$

$= 25,319.83 + 3000 = \boxed{28,319.83}$

3. A newborn child receives a $20,000 gift towards college education from her grandparents. How much will the $20,000 be worth in 17 years it is invested at 7% compounded quarterly?

One time Deposit ↳ F.V.

FV w/ Compound Interest

$$F = P(1+i)^n$$

$= \boxed{65,068.44}$

4. A company estimates that it will have to replace a piece of equipment at a cost of $10,000 in 5 years. The owner wants to have this money available when the equipment is replaced. He can make fixed quarterly payments and earn interest at 6% annual interest compounded quarterly. How much should the payments be?

~~Annuity~~ or Sinking Fund

$$E = \frac{Fi}{(1+i)^n - 1} = 10000 * 0.06/4 \Big/ \left((1+0.06/4)^{20} - 1\right)$$

$= \boxed{432.49}$

Poppers 1-5 A-E

5. Jenna wants to begin saving money for a new car. She can make monthly payments of $150 into an account at her credit union which pays 5% annual interest compounded monthly. How much money will she have available for her new car in three years?

Annuity

F.V.

$F = E\left(\left(1+i\right)^{n} - 1\right) / \left(i\right)$

$= 5,613$

6. The manager of a manufacturing company knows that they will need a new machine in one of their factories. The new machine will cost them $12,500. The manager has determined that they can afford to pay 20% of the cost of the machine in cash. They can then finance the rest through a credit union. The credit union will charge 2% per year compounded monthly. How much are their monthly payments for 4 years?

Amort or ~~Sinking~~

$P = 12500 - 20\%$

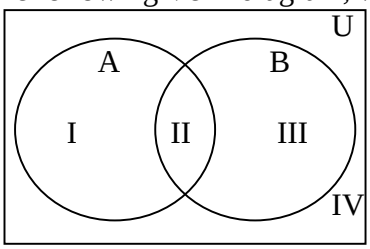
$= 12500 - 0.2(12500)$

$= 10,000$

$E = \dfrac{Pi}{1-(1+i)^{-n}}$

$= 216.95$

Example 7: Given the following Venn diagram, which region(s) make(s) up:



a. $A^c \cap B$ $\{III, IV\} \cap \{II, III\}$

In common

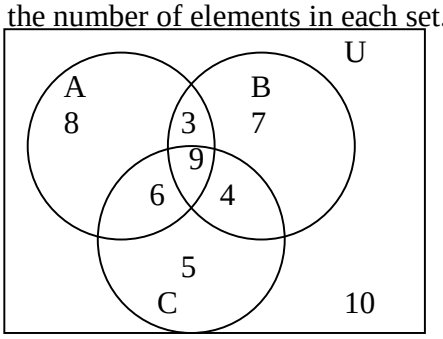
III

b. $A^c \cap B^c$ → $(A \cup B)^c$ = Outside the two circles

$\{III, IV\} \cap \{I, IV\}$ = IV

= IV

Example 8: Find the number of elements in each set.

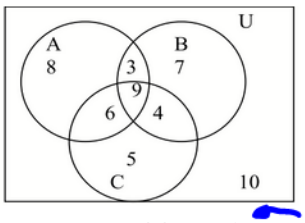


a. $n((B \cup C) \cap A^c)$

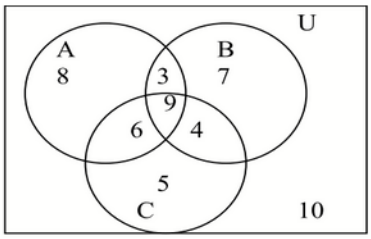
In common

7, 4, 5 ← Add them

16



b. $n(A^c \cup (C^c \cap B))^c$ → $A \cap (C^c \cap B)^c$ → Again → $A \cap (C \cup B^c)$

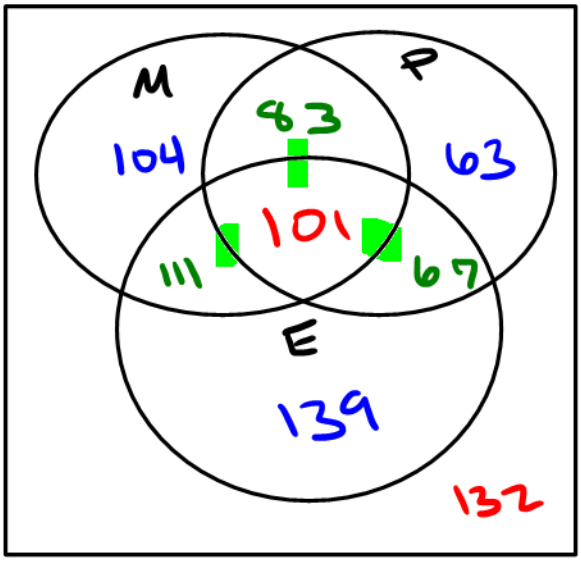


In common

8, 6, 9 = 23

Example 9: 800 college freshmen were surveyed regarding their enrollment in Math, Physics and English classes. The survey revealed the following:

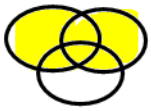
184 were enrolled in Physics and Math
399 were enrolled in Math
168 were enrolled in Physics and English
314 were enrolled in Physics
101 were enrolled in all three
212 were enrolled in Math and English
418 were enrolled in English



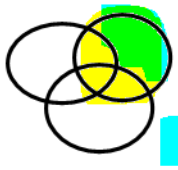
Total inside circles = 668

800 − 668 = 132

a. How many were enrolled in Math or Physics?

$M \cup P$  $= 529$

b. How many were not enrolled in Math or English but were enrolled in Physics?

$(M \cup E)^c \cap P$  $= 63$

Example 10: Suppose 4 pens are selected at random from a box containing 9 yellow pens and 6 blue pens. In how many ways can you choose at least 1 yellow pen? Use complement (0 yellow)

$n(S) = \text{Total stuff} = C(15,4) = 1365$

$E^c = \underset{0}{9Y} \quad \underset{4}{6B}$

$= C(9,0) \cdot C(6,4)$

$= 15$

$n(E) = n(S) - n(E^c)$

$= 1365 - 15$

$= 1350$

Example 11: A club has 58 members, 38 men and 20 women. A committee must consist of 8 people. In how many ways can the committee consist of at most 1 woman? 0, 1 W

$\underset{0}{20W} \quad \underset{8}{36M} \rightarrow C(20,0) \cdot C(36,8) = 44,903,492$

$1 \quad 7 \rightarrow C(20,1) \cdot C(36,7) = 252,405,120$

Add them

$= 301,308,612$

Example 12: A business organization needs to make up a 5 member fund-raising committee. The organization has 10 accounting majors and 8 finance majors. In how many ways can the fund-raising committee be formed if at most 1 accounting major is on the committee?

$\underset{0}{10\,Acct} \quad \underset{5}{8\,Fin} \rightarrow C(10,0) \cdot C(4,5) = 56$

$1 \quad 4 \rightarrow C(10,1) \cdot C(4,4) = 700$

756

Example 13: If a coin is tossed 14 times.

a. What is the probability that heads will come up exact1y 10 times?

$n(S) = 2^{\text{tosses}} = 2^{14} = 16,384$

E = 10 Heads

$P(E) = \frac{1001}{16,384}$

$n(E) = C(14,10) = 1001$

$= \boxed{0.0611}$

b. What is the probability that heads comes up at most 12 times?

$E = 0, 1, 2, \ldots, 11, 12$

$P(E) = 1 - P(E^c)$

$E^c = 13, 14$

$n(E^c) = C(14,13) + C(14,14) = 15$

$= 1 - \frac{15}{16,384} = \boxed{0.9991}$

Example 14: A box contains 20 computer chips, 5 of which are defective. Seven chips are taken out at random for testing. What is the probability that

a. at least 4 are defective? 4, 5 Def.

$n(S) = C(20,7) = 77,520$

5 Def	15 Good	
4	3	$\rightarrow C(5,4)\,C(15,3) = 2275$
5	2	$\rightarrow C(5,5)\,C(15,2) = 105$

> Add them

$= 2380$

$P(E) = \frac{2380}{77,520} = \boxed{0.0307}$

b. at most 4 are defective? 0, 1, 2, 3, 4 Def

E^c = 5 Def

5 Def	15 G	
5	2	$\rightarrow C(5,5) \cdot C(15,2) = 105$

$P(E) = 1 - P(E^c)$

$= 1 - \frac{105}{77,520} = \boxed{0.9986}$

Don't forget Perm → Title, Rank placing in order

Coin toss $C($ tosses , # of H / T $)$