Section 6.4: Use of Counting Techniques in Probability

Some of the problems we will work will have very large sample spaces or involve multiple events. In these cases, we will need to use the counting techniques from the chapter 5 to help solve the probability problems. In particular, we'll work with the multiplication principle and combinations.

Let S be a sample space and let E be any event. Then $P(E) = \frac{n(E)}{n(S)}$

Example 1: Consider the experiment of tossing a fair coin 12 times. $n(S) = 2^{\text{tosses}} = 2^{12} = 4096$

a. Find the probability that the coin lands heads exactly three times?

E = 3 Heads

C(12,3) = 220

$P(E) = \frac{220}{4096}$

$= \boxed{0.0537}$

b. Find the probability that the coin lands heads either 4 or 5 times.

E = 4 heads or 5 Heads

n(E) = C(12,4) + C(12,5)

= 495 + 792

= 1287

$P(E) = \frac{1287}{4096}$

$= \boxed{0.3142}$

c. Find the probability that the coin lands tails at least 11 times. 11 or more

E = 11 tails, 12 tails

n(E) = C(12,11) + C(12,12)

12 + 1 = 13

$P(E) = \frac{13}{4096}$

$= \boxed{0.00317}$

d. Find the probability that the coin lands heads at least twice. 2 or more

E = 2 Heads, 3 Heads, 4 heads .... 12 Heads

E^C = 0 Heads, 1 Head

$n(E^C) = C(12,0) + C(12,1) = 1 + 12 = 13$

$P(E) = 1 - P(E^C) = 1 - \frac{13}{4096}$

$= \boxed{0.9968}$

Popper 3: An unbiased coin is tossed 14 times. Find the probability that the coin lands heads exactly 8 times.

a. 0.2433
b. 0.5714
c. 0.1833
d. 3003

Example 2: From a group of 5 freshman, 6 sophomores 4 juniors and 3 seniors, what is the probability that a staff of 3 freshman, 3 sophomores, 2 juniors and 2 seniors will be selected for the yearbook staff. Assume that each student is equally likely to be chosen.

Total of 18 people
Staff of 10 people

$n(S) = C(18,10) = 43,758$

$C(5,3) \cdot C(6,3) \cdot C(4,2) \cdot C(3,2)$

$10 \cdot 20 \cdot 6 \cdot 3 = 3600$

$P(E) = \frac{3600}{43,758} = \boxed{0.0823}$

Example 3: Five cards are selected at random without replacement from a well shuffled deck of 52 playing cards.

$n(S) = C(52,5) = 2,598,960$

a. What is the probability that the two cards drawn are aces?

4 Aces / 49 Not Aces
2 / 3

$C(4,2) \cdot C(48,3) = 103,776$

$P(E) = \frac{103,776}{2,598,960} = 0.0401$

b. What is the probability that at least one of the cards a heart?

↳ Use Complement

E^c = 0 Hearts
13 Hearts / 39 Not Hearts
0 / 5

$C(13,0) \cdot C(39,5) = 575,757$

$P(E) = 1 - P(E^c)$

$P(E) = 1 - \frac{575,757}{2,598,960} = \boxed{0.7785}$

c. What is the probability that at least one of the cards is red?

E^c = 0 Red
26 Red / 26 Black
0 / 5

$C(26,0) \cdot C(26,5) = 65,780$

$P(E) = 1 - P(E^c)$

$= 1 - \frac{65,780}{2,598,960} = \boxed{0.9747}$

Note that the phrase "at least one" is a clue that you can use $P(E) = 1 - P(E^c)$

Popper 4: Six cards are selected at random without replacement from a well shuffled deck of 52 playing cards. What is the probability of getting at least 4 black cards? **Should you use the complement to solve this?**

a. Yes
b. No

Example 4: An urn contains 25 marbles of which 9 are green and 16 are blue. What is the probability that a person choosing 4 marbles at random will choose $n(S) = C(25,4) = 12{,}650$

a. Exactly 3 green and one blue?

9G 16B
3 1

$C(9,3) \cdot C(16,1) = 1344$

$P(E) = \frac{1344}{12{,}650}$

$= 0.1062$

b. At least 2 green marbles? 2, 3, 4 = E

$E^C = 0, 1$

9G 16B
0 4
1 3

$C(9,0) \cdot C(16,4) = 1820$
$C(9,1) \cdot C(16,3) = 5040$
$+ \quad 6860$

$P(E) = 1 - P(E^C)$

$= 1 - \frac{6860}{12{,}650}$

$= 0.4577$

c. At most 1 blue marble? Upto 1

E = 0, 1 Blue

16B 9G
0 4
1 3

$C(16,0) \cdot C(9,4) = 126$
$C(16,1) \cdot C(9,3) = 1344$
$+ \quad 1470$

$P(E) = \frac{1470}{12{,}650}$

$= 0.1162$

Example 5: A class contains 30 students, 18 girls and 12 boys. A group of 5 students is chosen at random from the class to make a presentation to the school board. What is the probability that the group making the presentation is made up of $n(S) = C(30,5) = 142{,}506$

a. All boys?

12B 18G
5 0

$C(12,5) \cdot C(18,0) = 792$

$P(E) = \frac{792}{142{,}506}$

$= 0.0056$

b. More girls than boys?

14G	12B	
3	2	$C(14,3)\cdot C(12,2) = 53856$
4	1	$C(14,4)\cdot C(12,1) = 36,720$
5	0	$C(14,5)\cdot C(12,0) = 8,568$

$> 99,144$

$$P(E) = \frac{99,144}{142,506} = \boxed{0.6957}$$

c. At least 1 boy? → Use Complement (0 Boys)

12B	14G
0	5

$C(12,0)\cdot C(14,5) = 8,568 \leftarrow E^C$

$$P(E) = 1 - P(E^C)$$

$$= 1 - \frac{8,568}{142,506} = \boxed{0.9399}$$

Popper 5: A fair coin is tossed 25 times. What is the probability that at most 22 heads occur?

a. 0.999922
b. 0.000078
c. 0.999990
d. 0.000069
e. 0.000010