

Section 6.3: Rules of Probability

Let S be a sample space, E and F are events of the experiment then,

1. $P(E) \geq 0$, for any event E .
2. $P(S) = 1$
3. If E and F are mutually exclusive, ($E \cap F = \emptyset$), then $P(E \cup F) = P(E) + P(F)$.
(Note that this property can be extended to a finite number of events.)
4. If E and F are not mutually exclusive, ($E \cap F \neq \emptyset$), then
 $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
(Note that this property can be extended to a finite number of events.)
5. (Rule of Complements) If E is an event and E^c denotes the complement of E then
 $P(E^c) = 1 - P(E)$.

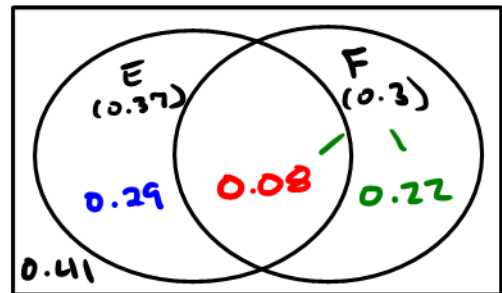
Example 1: An experiment consists of selecting a card at random from a well-shuffled deck of 52 playing cards. Find the probability that an ace or a spade is drawn.

$$\begin{aligned}
 P(\text{Ace} \cup \text{Spade}) &= P(\text{Ace}) + P(\text{Spade}) - P(\text{Ace} \cap \text{Spade}) \\
 &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} \approx \boxed{0.3077}
 \end{aligned}$$

Example 2: Let E and F be two events and suppose that $P(E) = 0.37, P(F) = 0.3$ and $P(E \cap F) = 0.08$. Compute

a. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$\begin{aligned}
 &= 0.37 + 0.3 - 0.08 \\
 &= \boxed{0.59}
 \end{aligned}$$



b. $P(E^c \cap F)$ outside of E AND in F
 "F only" $\{0.22, 0.41\} \cap \{0.08, 0.22\}$

$$= \boxed{0.22}$$

c. $P(E \cap F)^c = 1 - P(E \cap F)$

$$\begin{aligned}
 &= 1 - 0.08 \\
 &= \boxed{0.92}
 \end{aligned}$$

Popper 3: Let E and F be two events and suppose that $P(E) = 0.43, P(F) = 0.38$ and $P(E \cap F) = 0.08$. Find $P(E^c)$

- a. 0
- b. 0.43
- c. 0.62
- d. 0.57

Example 3: The SAT Math scores of a senior class at a high school are shown in the table.

Range of Scores	# of Students in the Range
$x > 700$	16
$600 < x \leq 700$	94
$500 < x \leq 600$	165
$400 < x \leq 500$	309
$300 < x \leq 400$	96
$x < 300$	20

Total # of students = 700

a. Construct the probability distribution for the data.

X	$x > 700$	$600 < x \leq 700$	$500 < x \leq 600$	$400 < x \leq 500$	$300 < x \leq 400$	$x < 300$
	16/700	94/700	165/700	309/700	96/700	20/700
P(X)	0.0229	0.1343	0.2357	0.4414	0.1371	0.0286

b. If a student is selected at random, what is the probability that his or her score was:

i. More than 500? $= 0.0229 + 0.1343 + 0.2357 = 0.3929$

or $\frac{16 + 94 + 165}{700}$

ii. Less than or equal to 400? $0.1371 + 0.0286 = 0.1657$

or $\frac{96 + 20}{700}$

iii. Greater than 400 but less than or equal to 700?

$0.1343 + 0.2357 + 0.4414 = 0.8114$

or $\frac{94 + 165 + 309}{700}$

Pepper 1 is B

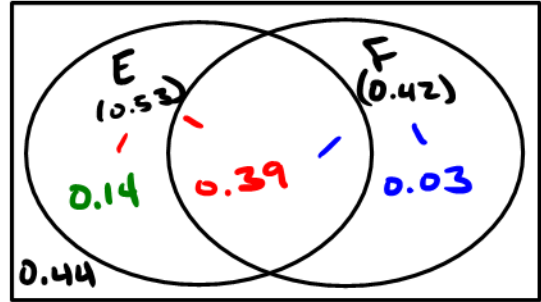
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Example 4: Let E and F be events of a sample space S. Let $P(E) = 0.53$, $P(F) = 0.42$ and $P(E \cap F^c) = 0.14$. ← E only

a. Find $P(E \cup F)$ = Total in circles

$$= 0.14 + 0.39 + 0.03$$

$$= \boxed{0.56}$$



b. Find $P(E^c \cup F^c)$ = DeMorgan's Law

$$P(E \cap F)^c = 1 - P(E \cap F)$$

$$= 1 - 0.39 = \boxed{0.61}$$

c. Find $P(E \cup F)^c$ ← Outside the circles = $\boxed{0.44}$

$$= 1 - P(E \cup F)$$

$$= 1 - 0.56 =$$

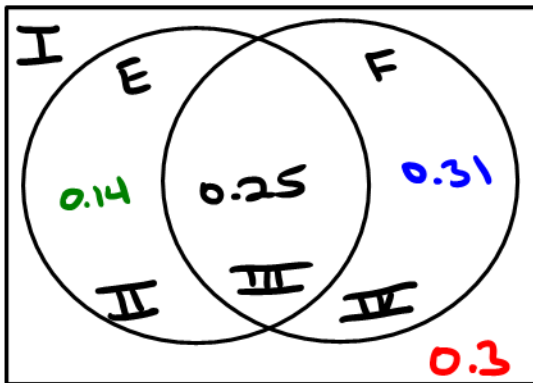
d. $P(E \cup F^c)^c = P(E^c \cap F)$ = Outside of E AND part of F

$$= F \text{ only}$$

$$= \boxed{0.03}$$

Example 5: Let E and F be events of a sample space S. Let $P(E^c) = 0.61$, $P(F^c) = 0.44$ and $P(E \cup F)^c = 0.3$. Find $P(E^c \cup F)^c$.

Outside the Circles = I



$P(E^c) = I + IV = 0.61$

$0.3 + IV = 0.61$

$IV = 0.31$

$I + II + III + IV = 1$

$III = 0.25$

$P(F^c) = I + II = 0.44$

$0.3 + II = 0.44$

$II = 0.14$

$P(E^c \cup F)^c = P(E \cap F^c)$

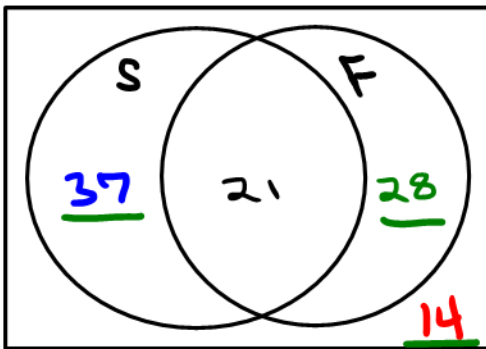
E only

$= 0.14$

Popper 4: Suppose $P(E^c) = 0.21, P(F) = 0.76, P(E^c \cap F) = 0.18$.
Find $P(E \cup F)^c$

- a. 0.58
- b. 0.21
- c. 0.03
- d. 0.24

Example 6: One-hundred students were asked if they speak Spanish or French. It was found that 37 speak only Spanish, 14 speak none of these languages, and 79 don't speak both languages. If a student is picked at random, what is the probability that he or she can speak French only?



$$79 = 37 + x + 14 \Rightarrow n(S \cap F)^c$$

$$28 = x$$

$$P(\text{French only}) = \frac{28}{100}$$

$$= \boxed{0.28}$$

Example 7: An experiment consists of selecting a card at random from a well-shuffled deck of 52 playing cards. Find the probability that

- a. a spade or a queen is drawn.

$$P(\text{Spade} \cup \text{Queen}) = P(\text{Spade}) + P(\text{Queen}) - P(\text{Spade} \cap \text{Queen})$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{0.3077}$$

- b. a 7 or an 8 is drawn.

$$P(7 \cup 8) = P(7) + P(8) - P(7 \cap 8)$$

$$= \frac{4}{52} + \frac{4}{52} - \frac{0}{52} = \frac{8}{52} = \boxed{0.1538}$$

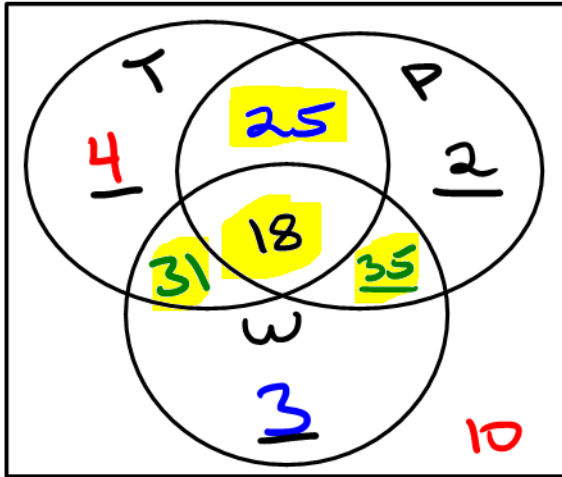
- c. a red card or a face card?

$$P(\text{Red} \cup \text{Face}) = P(\text{Red}) + P(\text{Face}) - P(\text{Red} \cap \text{Face})$$

$$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$$

$$= \frac{32}{52} = \boxed{0.6154}$$

Example 8: You are a chief for an electric utility company. The employees in your section cut down trees, climb poles, and splice wire. You report that of the 128 employees in your department 10 cannot do any of the three (management trainees), 25 can cut trees and climb poles only, 31 can cut trees and splice wire, but not climb poles, 18 can do all three, 4 can cut trees only, 3 can splice wire, but cannot cut trees or climb poles, and 9 can do exactly one of the three.



$$4 + 25 + 2 + 18 + 31 + 3 + 10 = 93$$

$$128 - 93 = 35$$

One of these employees is selected at random. What is the probability that that employee

a. can do at exactly one of the three jobs mentioned here?

$$\frac{4 + 2 + 3}{128} = \frac{9}{128} = \boxed{0.0703}$$

b. can cut trees and climb poles?

$$\frac{25 + 18}{128} = \frac{43}{128} = \boxed{0.3359}$$

c. at least two of the jobs mentioned here?

$$\begin{aligned}
 2 \text{ Jobs} &= 25 + 31 + 35 \\
 3 \text{ Jobs} &= 18
 \end{aligned}
 \quad \begin{array}{l} > \\ \text{Add} \end{array} = 109$$

$$\frac{109}{128} = \boxed{0.8516}$$

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Popper 5: Suppose $P(E^c) = 0.21$, $P(F) = 0.76$, $P(E^c \cap F) = 0.18$.
Find $P(E \cap F)^c$

- a. 0.03
- b. 0.39
- c. 0.42
- d. 0.58