AP Calculus BC Course Syllabus

Resources

Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. Calculus. Lexington, Massachusetts: D.C. Heath and Company, 5th edition 1994 – student issued textbook

Hughes-Hallett, Deborah, et al. Calculus – Single Variable. New York: John Wiley and Sons, Inc.

Calculus for a New Century by Fraga

Stewart, James. Calculus. Pacific Grove, CA: Brooks/Cole

Finney, Ross L., et al. Calculus: Graphical, Numerical, Algebraic: AP Edition. Boston Wesley.

Ostobee, Arnold, and Paul Zorn. Calculus from Graphical, Numerical, and Symbolic View. Boston: Houghton Mifflin.

Anton, Howard. Calculus: A new Horizon. New York: John Wiley & Sons

Assessments:

Major topics will have at least one major test. This generally works out to a major test for 10 days of lessons. Quizzes will be used frequently to assess current knowledge and prepare for major assessments.

Course Description

Calculus BC is designed to develop the understanding of the concepts of calculus and provide experience with its methods and applications. The course represents a multi-representational approach to calculus, with concepts, results and problems expressed geometrically, numerically, analytically and verbally. The connections among these representations are important.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems or problem types. Technology should be used regularly to reinforce the relationships among multiple representations of functions, to confirm written work, to implement experimentation and to assist in interpreting results.

Through the use of unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all functions from Pre-Calculus, such as those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise defined.

Content

Analysis of graphs (all year): With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function, verbally and orally.

Limits of functions: (including one-sided limits) (10 days)

- demonstrate an intuitive understanding of the limiting process by estimating limits from graphs or tables of data
- determine the limit as $x \to c$ of a sum, difference, product, and quotient of two or more functions, and verify using technology
- evaluate limits using algebraic techniques
- evaluate one-sided limits and describe the concept of continuity in terms of limits
- describe the asymptotic behavior of functions in terms of limits involving infinity

Continuity as a property of functions (4 days)

- develop an intuitive understanding of continuity
- understanding continuity in terms of limits
- geometric understanding of graphs of continuous functions including the Intermediate Value Theorem
- analyze planar curves including those given in parametric form, polar form, and vector form

Concept of the derivative (2 days)

- present and analyze the derivative graphically, numerically and analytically
- define the derivative of a function as the limit of the difference quotient
- use the limit process to find the derivative of a function
- explain the relationship verbally, graphically and in written form between differentiability and continuity

Computation of the derivative and Derivative at a point (20 days)

- develop and apply the rules for determining derivatives of functions including constant, power, trigonometric, inverse trigonometric, exponential and logarithmic functions
- apply basic rules for derivatives of sums, products and quotients of functions
- determine the derivative of composite functions by applying the chain rule
- determine the derivative using implicit differentiation
- determine derivatives of parametric, polar, and vector functions
- analyze the derivative as a slope of a curve at a point, including points at which there are vertical tangents and points at which there are no tangents.
- apply the derivative at a point to determine the tangent and normal lines to a curve and local linear approximation
- verbally define and apply instantaneous rate of change as the limit of average rate of change to solve problems
- approximate rate of change from graphs and tables of values
- find the value of the derivative at a point using technology

Derivative as a function (5 days)

- understand and verbalize the corresponding characteristics of the graphs of a function f and its first derivative f'
- observe the relationship between the increasing and decreasing behavior of f and the sign of f'
- apply Mean Value and Rolle's Theorems and identify their geometric consequences
- translate descriptions of derivatives into equations and vice versa
- describe how to find relative extrema, orally and in writing

Second derivative (2 days)

- recognize and describe in writing the relationship between the concavity of a function and the sign of its second derivative
- recognize the corresponding characteristics of the graphs of $f, f' \text{ and } f''$, and be able to explain verbally
- recognize points of inflections as places where concavity changes
- graph the derivative with technology and use it to describe the second derivative verbally

Applications of derivatives (14 days)

- analyze curves, including the notions of monotonicity and concavity
- analyze planar curve given in parametric form, polar form, and vector form, including velocity and acceleration vectors
- make and use informal sketches to illustrate the situation given a related rates problem
- model rates of change, including related rates problems
- determine the reasonableness of solutions to related rates problems, including sign, size, relative accuracy, and units of measurement and write an accurate conclusion statement
- find extrema of a function including relative and absolute extrema utilizing graphical and numerical techniques
- apply the first and second derivative tests to determine relative extrema, and write an accurate conclusion statement
- given real world and purely mathematical situations, apply derivatives to solve optimization problems, considering both absolute and relative extrema, algebraically and verifying graphically
- interpret the derivative as a rate of change in varied applied contexts including velocity, speed and acceleration
- use implicit differentiation to find the derivative of an inverse function
- geometrical interpretation of differentials equations via slope fields and the relationship between slope fields and solution curves for differential equations
- use Euler's method to approximate the numerical solution of differential equations
- apply L'Hopital's Rule, including its use in determining limits and convergence of improper integrals and series

Interpretations and properties of definite integrals (3 days)

- compute Riemann sums using left, right and midpoint evaluation points
- interpret the definite integral as a limit of Riemann sums over equal subdivisions

- interpret the definite integral of the rate of change of a quantity over an interval to be the change of quantity over the interval $\int_a^b f'(x)dx = f(a) - f(b)$
- use basic properties of definite integrals, including additivity and linearity

Applications of integrals (20 days)

- use the integral of a rate of change to accumulate change
- set up an approximating Riemann sum and represent its limit as a definite integral
- find specific anti-derivatives using initial conditions
- find the area under a given curve
- find the area of a region, including a region bounded by polar curves
- determine approximate area under a given curve using the Trapezoidal Rule
- apply The Mean Value Theorem for integrals and its geometric consequences to find the average value of a function, and explain its meaning
- understand the use of the definite integral to define a function
- apply integration techniques to find the volume of a solid with known cross sections
- apply integration techniques to find the volume of a solid of a revolution using both shell and disc methods
- use the definite integrals to find displacement and distance traveled by a particle along a line in a specific time period from a given velocity or acceleration function
- find the length of a curve, including a curve given in parametric form
- apply integration techniques to model physical, biological or economic situations and write a complete and accurate conclusion statement
- use technology to approximate the value of a definite integral

Fundamental Theorem of Calculus (6 days)

- apply the Fundamental Theorem of Calculus to evaluate definite integrals
- use the Fundamental Theorem of Calculus to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
- apply and describe results from the Second Fundamental Theorem of Calculus

Techniques of antidifferentiation (12 days)

- identify and use antiderivatives following directly from derivatives of basic functions
- find antiderivatives by substitutions of variables (including change of limits for definite integrals)
- find antiderivatives by parts, and by simple partial fractions (non-repeating linear factors only)
- determine improper integrals (as limits of definite integrals)

Applications of Antidifferentiation (6 days)

- find specific antiderivatives using initial conditions, including motion along a line
- solve separable differential equations and use them in modeling
- study the equation $y' = ky$ and exponential growth
- solve logistic differential equations and use them in modeling

Numerical approximations to definite integrals (2 days)

- use Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Concept of series (4 days)

- define a series as a sequence of partial sums
- define convergence in terms of the limit of the sequence of partial sums
- use technology to explore convergence or divergence

Series of constants (10 days)

- describe series using motivating examples, including decimal expansion
- recognize and use geometric series with applications
- recognize and use the harmonic series
- recognize and use the alternating series with error bound
- recognize and use the terms of series as areas of rectangles and relate them to improper integrals, including the integral test and its use in testing the convergence of p-series
- recognize and use the ratio test for convergence and divergence
- compare series to test for convergence and divergence
- explain in writing what the error of an alternating series means

Taylor series (20 days)

- on the graphing calculator, experiment with multiple terms of a Taylor Polynomial to approximate various functions
- use graphical demonstrations of convergence to illustrate Taylor polynomial approximation
- recognize and use Maclaurin series and the general Taylor series centered at x – a
- recognize and use Maclaurin series for the functions e^x, *sin* x, *cos* x and 1/ (1 – x)
- use formal manipulation of Taylor series and shortcuts to compute Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
- determine functions defined by power series
- find radius and interval of convergence of power series
- apply Lagrange error bound for Taylor polynomials