Number Theory Vocabulary
(For Middle School Teachers)

A

- **Absolute value** – the absolute value of a real number is its distance from zero on the number line. The absolute value of any real number, a, written as $|a|$, is defined as: $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$.

- **Algorithm** – a step by step, recursive computational procedure for solving a problem in a finite number of steps.

- **Associative property of addition** – a set S satisfies this property if for all elements a, b, and c in S, (a + b) + c = a + (b + c), i.e. changing the grouping of the addends does not change the sum.
  - For more info: http://www.cleveland.k12.oh.us/Parents/Math/sld003.htm

- **Associative property of multiplication** – a set S satisfies this property if for all elements a, b, and c in S, (a • b) • c = a • (b • c), i.e. changing the grouping of factors does not change the product.
  - For more info: http://www.cleveland.k12.oh.us/Parents/Math/sld012.htm

B

- **Base 10 (decimal) system** – a counting system that is based on the number 10 and uses the digits 0, 1, 2… 9. The main principle of the decimal system is that 10 is considered as a new unit from which point counting starts again. The number 235 written in base 10 represents the value $2\times10^2 + 3\times10^1 + 5\times10^0$.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=421

C

- **Cardinal numbers** – a whole number that is used to specify the number of elements in a set.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=61

- **Closure of a set** – it is the smallest closed set containing the given set. A set is closed if its complement is open, where set is open if it is a neighborhood of all its points. For example: consider the open interval (0, 1). The closure of this set is [0, 1] which is the smallest closed set that contains (0, 1).

- **Combination** – The number of ways of picking *k unordered* outcomes from *n* possibilities. The number of combinations of *n* distinct objects taken *r* at a time is $C(n,r) = \frac{n!}{(n-r)!r!}$. Example: The number of ways of selecting a committee of 2 people out of 4 people is $C(4,2) = \frac{4!}{(4-2)!2!} = 6\ ways$. Let these people be denoted as A, B, C and D. Then the set of all possible combinations is {AB, AC, AD, BC, BD, CD}. Note that the choice AB is equivalent to BA, i.e. ordering doesn't matter.
  - For more info: http://www.pballew.net/permute.html

- **Common factor –** in arithmetic, a number that is a factor of two or more numbers. For example, 10 is a common factor of 70 and 100. In algebra, a polynomial that is a factor of two or more polynomials. That is, each of the polynomials is divisible by the common factor. For example, x – 1 is a common factor of $x^2 + x - 2$ and $x^2 - 6x + 5$, since each polynomial is divisible by x – 1.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=76

- **Commutative property of addition –** A set S satisfies the commutative property of addition if for all a and b in S, a + b = b + a. i.e. changing the order of the addends does not change the sum.
  - For more info: http://www.cleveland.k12.oh.us/Parents/Math/sld004.htm

- **Commutative property of multiplication –** A set S satisfies the commutative property of multiplication if for all a and b in S, a • b = b • a. i.e. changing the order of the factors does not change the product.
  - For more info: http://www.cleveland.k12.oh.us/Parents/Math/sld013.htm

- **Complex number** – a number of the form $a + b\boldsymbol{i}$, where *a* and *b* are real numbers and ***i*** is the imaginary unit equal to the square root of -1; *a* is called the real part, and *b* is called the imaginary part.

- **Complex plane** – a coordinate plane used for graphing complex numbers. The horizontal axis is the real axis and the vertical axis is the imaginary axis.
  - For more info: http://www.mathwords.com/c/complex_plane.htm

- **Composite number** – A positive number which has factors other than 1 and itself. Example: 6 has factors 2 and 3, other than 1 and 6.

- **Conjugate of complex number** – For any complex number z= a+bi, its conjugate $\bar{z}$ is equal to a-bi. In the complex plane the points representing a complex number and its conjugate are mirror images with respect to the real axis. Example: the conjugate of 2+3i is 2-3i.

- **Coordinate plane** – A two-dimensional region determined by a pair of axes.

D

- **Decimal expansion** – The decimal form of a rational number can be obtained by dividing the denominator into the numerator. The result will either be a terminating decimal or a repeating decimal. Example: 3/4= .75 and 24/11= $2.181818\ldots = 2.\overline{18}$.

- **Decimal fraction** – a fraction in which the denominator is any power of 10. For example, 3/10, 51/100, and 23/1,000 are decimal fractions and are normally expressed as 0.3, 0.51, and 0.023.

- **Density of number line** – between any two real numbers there is always another real number.

- **Distributive property of multiplication over addition** – this property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. For example, for all real numbers *a*, *b*, and *c*, $a \cdot (b+c) = a \cdot b + a \cdot c$ and $(b+c) \cdot a = b \cdot a + c \cdot a$
  - For more info: http://www.harcourtschool.com/glossary/mathadvantage/definitions/distributivep7.html

- **Divisibility rule for 2** – a numbers is divisible by 2 if its ones digit is divisible by 2. Thus for a number to be divisible by 2, the ones digit must be 0, 2, 4, 6 or 8.

- **Divisibility rule for 3** – a number is divisible by 3 if the sum of all its individual digits is divisible by 3. For example, the sum of the digits for the number 3627 is 18, which is divisible by 3 so the number 3627 is divisible by 3.

- **Divisibility rule for 4** – a number is divisible by 4 if the number formed by the last two individual digits is divisible by 4. For example, the number formed by the last two digits of the number 3628 is 28, which is divisible by 4 so the number 3628 is evenly divisible by 4.

- **Divisibility rule for 5** – a number is divisible by 5 if its last digit is either 0 or 5.

- **Divisibility rule for 6** – a number is divisible by 6 if it's divisible by both 2 AND 3. For example: 23160 is divisible by 6 since it's divisible by 2 and the sum of its digits is 12, which is divisible by 3. The number is divisible by both 2 and 3 and hence divisible by 6.

- **Divisibility rule for 7** – to determine if a number is divisible by 7, take the last digit off the number, double it and subtract the doubled number from the remaining number. If the result

is divisible by 7 (e.g. 14, 7, 0, -7, etc.), then the original number is divisible by seven. This may need to be repeated several times.

- **Divisibility rule for 8** – a number is divisible by 8 if the number formed by the last three digits is divisible by 8. For example, the number formed by the last three digits of 3624 is 624, which is divisible by 8 so 3624 is divisible by 8.

- **Divisibility rule for 9** – a number is divisible by 9 if the sum of all its individual digits is divisible by 9. For example, the sum of the digits of the number 3627 is 18, which is evenly divisible by 9 so 3627 is evenly divisible by 9.

- **Divisibility rule for 10** – a number is divisible by 10 only if 0 is in the units place.

- **Divisibility rule for 11** – if the difference between the sum of the odd-numbered digits and the sum of the even numbered digits, counted from right to left, is divisible by 11, then the number is divisible by 11.

  For example, consider the number 528:
  8 + 5 = 13 (the sum of odd-numbered digits)
  2 (the sum of even-numbered digits)
  13 - 2 = 11 and 11 is divisible by 11, therefore 528 is divisible by 11.

- **Divisibility rule for 12** – if the number is divisible by both 3 and 4, it is also divisible by 12.

- **Divisor** –An integer *a* is a divisor of an integer *b* if there exists an integer c such that $ac=b$.

E

- **Equivalence properties of equality** – The reflexive (a=a), symmetric (if a=b then b=a) and transitive (a=b, b=c then a=c) satisfied by the "=" relation.
  - For more info:
    http://www.mathwords.com/e/equivalence_properties_of_equality.htm

- **Estimation** – an approximation for the result of a calculation or a guess about the size, cost, or quality of something.
  - For more info:
    http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=136

- **Even number** – An even number is an integer that is divisible by 2; thus, all even numbers can be written in the form $2n$, where n is an integer.

- **Exponent** – An exponent is the power p in an expression of the form a^p. For example, 3 is the exponent and 9 the base in the expression 9^3.

F

- **Factor** – an integer *a* is said to be a factor of an integer *b*, if there exists an integer *c* such that *ac*=*b*. For example, 6 is a factor of 42, 6 is not a factor of 44.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=146

- **Finite set** – A set that contains exactly n elements, where n is a natural number.

G

- **Greatest common divisor/greatest common factor** (GCD or GCF) – the greatest common divisor of two positive integers is the largest divisor common to both. For example, 16 is the GCF of 32 and 48.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=160

I

- **Identity element of addition** – e is an additive identity for a set S if $a+e=a=e+a$ for all a in S. 0 is the unique additive identity for the reals.

- **Identity element of multiplication** – e is a multiplicative identity for a set S if $a.e=a=e.a$ for all a in S. 1 is the unique multiplicative identity for the reals.

- **Identity property of addition** – The sum of any number *a* and zero is *a*.
  - For more info: http://www.cleveland.k12.oh.us/Parents/Math/sld005.htm

- **Identity property of multiplication –** the product of any number *a* and 1 is *a.*
  - For more info: http://www.cleveland.k12.oh.us/Parents/Math/sld014.htm

- **Imaginary number** – a number of the form ***bi***, where b is a real number and b ≠ 0 and $i=\sqrt{(-1)}$.
  - For more info: http://www.mathwords.com/i/imaginary_numbers.htm

- **Improper fraction** – a fraction whose numerator is greater than (or equal to) its denominator. For example, 22/7 is an improper fraction.

- **Infinite set** – a set which doesn't have a finite cardinality. The set of natural numbers is the smallest infinite set.

- **Integer** – an element that belongs to the union of the set of positive whole numbers, their additive inverse (b is an additive inverse of "a" if a+b=0=b+a) and 0. For example: -3, 0, 1 are integers.

- **Integer part** –the largest integer less than or equal to x. For example: [2.8]=2 , [-2.1]= -3. Denoted as [x].

- **Inverse element of addition** – An element a' is the additive inverse of the element a, if $a + a' = 0 = a' + a.$ i.e. a number plus its additive inverse equals the additive identity 0. For example, the additive inverse of 5 is -5.
  - For more info: http://www.oswego.org/mtestprep/math8/b/add_inv.cfm

- **Inverse element of multiplication** – An element a' is the multiplicative inverse of an element a, if $a.a' = 1 = a'.a$. For example, the multiplicative inverse of 3 is 1/3.
  - For more info: http://www.oswego.org/mtestprep/math8/b/mult_inv.cfm

- **Inverse property of addition –** The sum of a number and its additive inverse is equal to 0 (the additive identity).For every real number *a* there exists another real number *-a,* such that *a + (-a) = 0.*

- **Inverse property of multiplication** – The product of a number and its multiplicative inverse is equal to1 (the multiplicative identity). For every non zero real number *a,* there exists a real number a^{-1}, such that $a.a^{-1} = 1$.

- **Irrational number** – irrational numbers are real numbers that cannot be expressed as a ratio of two integers; thus, cannot be written as terminating or repeating decimals.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=185

L

- **Least common multiple (LCM)** – the least common multiple of a and b, denoted lcm (a, b), is the smallest positive number that is a multiple of both a and b.
  For example, 60=lcm (12, 10).
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=192

M

- **Mental math** – Performing computations mentally, without writing anything down.

- **Mixed number** – a number combining a whole number with a fraction.
  For example, $2\frac{7}{8}$ is a mixed number.

- **Multiple** – An integer *b* is a multiple of an integer *a*, if there exists an integer *c* such that *ac=b*. For example 20 is a multiple of 4 since there exists the integer 5, such that 4.5=20

N

- **Natural number** – an element of the set {1, 2, 3, 4…}. The set of natural numbers is the same as the set of positive integers and counting numbers.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=224

- **Non-repeating Non terminating decimal** – a decimal that neither terminates nor repeats. It represents an irrational number. For example, 3.14159265359…… is a non-repeating/non terminating decimal.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=228

- **Number line** – a line with a fixed scale so that every real number corresponds to a unique point on the line.
  - For more info: http://mathworld.wolfram.com/RealLine.html

- **Number theory** – is the mathematical study of integers and their generalizations.

- **Numeration system** – the writing or stating numbers in a particular order. The Romans used I, II, IX etc, the Greeks used $\alpha, \beta ...$ etc and the Hindu-Arabic number system is the one which is currently used today.

O

- **Odd number** – An integer that is not divisible by 2; thus, all odd numbers can be written in the form $2n + 1$, where n is an integer. Odd numbers have 1, 3, 5, 7 or 9 in the units' place.

- **Operation** – a mathematical procedure such as addition, subtraction, multiplication, and division applied to elements of a set.

P

- **Percent** – per hundred. It is the ratio of a number to 100.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=253

- **Perfect square** – a number that is the square of a whole number.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=255

- **Permutation** – a permutation of n objects can be thought of as all the possible ways of their arrangement or rearrangement. The number of 'permutations of n objects taken r at a time' is denoted by ${}^{n}P_{r}$ $which\,equals$ $\frac{n!}{(n-r)!}$. For example there are 6 permutations of A, B, C taken two at a time: AB, AC, BA, BC, CA, and CB.

- For more info:
  http://www.mathwords.com/p/permutation.htm

- **Place value** – Place value means that each position in a number has a different value associated with it, that is, 362 is three 100s, six 10s, and two 1s.
  - For more info:
    http://www.ex.ac.uk/cimt/mepres/book7/bk7i2/bk7_2i1.htm

- **Polynomial** – A function of the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... a_1 x + a_0$ for all real x, where the coefficients are real numbers and n a non negative integer. If $a_n \neq 0$, P(x) is called a real polynomial of degree n.
  - For more info:
    http://www.purplemath.com/modules/polydefs.htm

- **Prime factorization** – the expression of a composite number as a product of prime numbers. For example, $70 = 2 \cdot 5 \cdot 7$.
  - For more info:
    http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=272

- **Prime number –** a positive integer greater than 1 with exactly two positive divisors: 1 and itself. For example, 2, 3, 5, 7 are all prime numbers.

- **Proper fraction** – a fraction whose numerator is less than its denominator.

- **Pythagorean Theorem** – gives the relationship between the lengths of the sides of a right angled triangle. The theorem states: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  - For more info:
    http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=281

Q

- **Qualitative reasoning** – the application of reasoning based on non-numerical data such as problem solving, reasoning skills, and communication.

- **Quantitative reasoning –** the application of reasoning based on numerical data such as graphs and tables.

R

- **Rational number** – a number that can be expressed as $\frac{m}{n}$, where *m* and *n* are integers and *n* is not zero.
  - For more info:
    http://mathworld.wolfram.com/RationalNumber.html

- **Real numbers** – the set of irrational numbers together with the set of rational numbers.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=299

- **Rectangular array** – a set of elements arranged into rows and columns such as a matrix.
  - For more info: http://www.mathwords.com/m/matrix.htm

- **Remainder –** The term is most commonly used to refer to the number left over when two integers are divided by each other in integer division. For integers a and b, with b>0, there exist unique integers q and r such that a=bq+r, where $0 \le r < b$. The number r is called the remainder.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=410

- **Repeating decimals** – a decimal number that repeats a pattern of digits continuously on the right. For example, 6.14141414…. is a repeating decimal.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=309

- **Root** – Let f(x) =0 be an equation that involves the indeterminate x. A root of the equation is a value c such that f(c) =0.

S

- **Scientific notation** – a way to express a very large or very small number as the product of a number between 1 and 10 and a power of 10. For example, the scientific notation for 2,500,000 is $2.5 \cdot 10^6$.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=321

T

- **Terminating decimals** – a decimal with a finite number of digits, i.e. the sequence of digits after the decimal point does not continue forever. For example, ¼ = 0.25 is a terminating decimal.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=357

V

- **Venn diagram** – A method of displaying relations between subsets of some universal set. The universal set E is represented by the interior of a rectangle, and the subsets of E are represented by regions inside this by simple closed curves.
  - For more info: http://www.venndiagram.com

W

- **Whole number** – an element of the set {0, 1, 2, 3, 4…….}. The set of whole numbers is the same as the set of non negative integers.
  - For more info: http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=386