

How Much Does a Grecian *Urn*?



Urn 1



Urn 2

A Grecian gentleman has the opportunity to *urn* a prize. He is given two urns and four marbles, two red and two white marbles. He may arrange the marbles in the two urns any way he chooses. Then another person will randomly draw one marble out of one of the urns. If the marble is white, the Grecian gentleman will *urn* the prize. If the marble is red, he will *urn* nothing.

The Grecian gentleman wants your help. How could he best arrange the four marbles in the two urns to improve his chances of *urning* the prize?

Questions

The Grecian gentleman wants your help.

1. How could he best arrange the four marbles in the two urns to improve his chances of *urning* the prize?
2. Suppose he is required to put at least one marble in each of the urns. Will this affect how he should arrange the four marbles? Why or why not?
3. Repeat the problem, this time with 3 white and 2 red marbles.
4. Repeat the problem, this time with 2 white and 3 red marbles.
5. Repeat the problem, this time with 3 urns, and 3 white and 3 red marbles.
6. Repeat the problem, this time with 3 urns, and 2 white and 4 red marbles.
7. State and prove a conjecture concerning the arrangement that will maximize your chances of *urning* a prize.

Area Model for Determining the P(W)

Case 2 example

| | | |
|--------------------|--------------|-----|
| Urn 1 1/2 | Urn 2 1/2 | |
| 1 W Area 1/2 | 1 W 1/6 | 1/3 |
| | 1 R 1/6 | 1/3 |
| | 1 R 1/6 | 1/3 |
| 1/2 | 1/2 | |

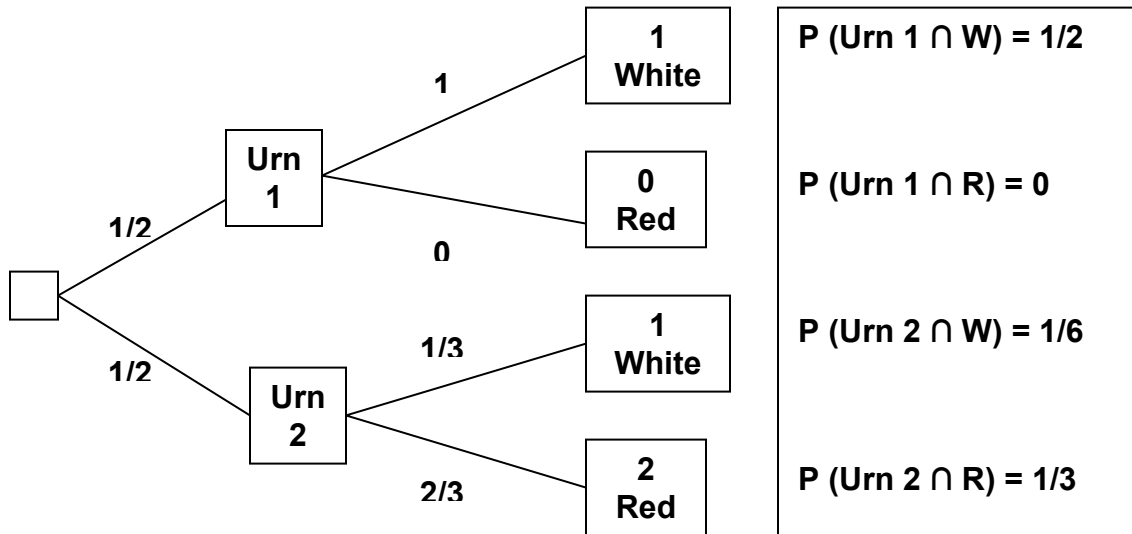
$$P(W) = P(\text{Urn 1 and W}) + P(\text{Urn 2 and W}) = 1/2 + 1/6 = 2/3$$

Case 3 example

| | | |
|--------------------|--------------------|-----|
| Urn 1 1/2 | Urn 2 1/2 | |
| 1 W area 1/4 | 1 W area 1/4 | 1/2 |
| 1 R area 1/4 | 1 R area 1/4 | 1/2 |
| 1/2 | 1/2 | |

$$P(W) = P(\text{Urn 1 and W}) + P(\text{Urn 2 and W}) = 1/4 + 1/4 = 1/2$$

Tree Diagram of One of the Best Solutions



$$\begin{aligned} P(\text{winning}) &= P(\text{Urn 1} \cap \text{white}) + P(\text{Urn 2} \cap \text{white}) \\ &= 1/2 + 1/6 = 3/6 + 1/6 = 4/6 \text{ or } 2/3 \end{aligned}$$

Solution Possibilities

| Case | Urn 1 | Urn 2 | P(white) | P(red) |
|-----------------------------------|---------------|---------------|------------|------------|
| Case 1: 4 marbles in Urn 1 | 2W, 2R | empty | 1/2 | 1/2 |
| Case 2: 3 marbles in Urn 1 | 2W, 1R | 1R | 1/3 | 2/3 |
| Case 2: 3 marbles in Urn 1 | 1W, 2R | 1W | 2/3 | 1/3 |
| Case 3: 2 marbles in Urn 1 | 2W | 2R | 1/2 | 1/2 |
| Case 3: 2 marbles in Urn 1 | 2R | 2W | 1/2 | 1/2 |
| Case 3: 2 marbles in Urn 1 | 1W, 1R | 1W, 1R | 1/2 | 1/2 |
| Case 4: 1 marble in Urn 1 | 1R | 2W, 1R | 1/3 | 2/3 |
| Case 4: 1 marble in Urn 1 | 1W | 1W, 2R | 2/3 | 1/3 |
| Case 5: 0 marbles in Urn 1 | empty | 2W, 2R | 1/2 | 1/2 |