How Much Does a Grecian *Urn*?

Purpose:
Participants will determine the best placement of the marbles to increase their chances of winning the prize.

Overview:
In small groups, participants will investigate various placements of the two red and two white marbles in the two urns. They will determine the probability of drawing each color marble for each arrangement in order to determine the best placement of the marbles which will increase their odds of *urning* a prize.

TExES Mathematics 4-8 Competencies. The beginning teacher:

IV.013.A	Explores concepts of probability through data collection, experiments, and simulations.
IV.013.B	Uses the concepts and principles of probability to describe the outcome of simple and compound events.
IV.013.D	Determines probabilities by constructing sample spaces to model situations.

TEKS Mathematics Objectives. The student is expected to:

4.13.A	List all possible outcomes of a probability experiment such as tossing a coin.
5.12.A	Use fractions to describe the results of an experiment.
5.12.B	Use experimental results to make predictions.
6.9.A	Construct sample spaces using lists, tree diagrams, and combinations.
7.10.A	Construct sample spaces for compound events (dependent and independent).
7.11.B	Make inferences and convincing arguments based on an analysis of given or collected data.
8.11.A	Find the probabilities of compound events (dependent and independent).
8.11.B	Use theoretical probabilities and experimental results to make predictions and decisions.

Terms.
Probability, deterministic vs. random experiment, sample space, finite vs. infinite sample space,

Materials.
For each pair of participants:

- Two urns (containers, boxes, bags)
- Four red and four white marbles or cubes
- Transparencies and Activity Sheets

Transparencies.

- *Transparency: How Much Does a Grecian Urn?*
- *Transparency: Solution Possibilities*
- *Transparency: Area Model*
- *Transparency: Tree Diagram*

Activity Sheet(s).

- *Activity Sheet: How Much Does a Grecian Urn?*

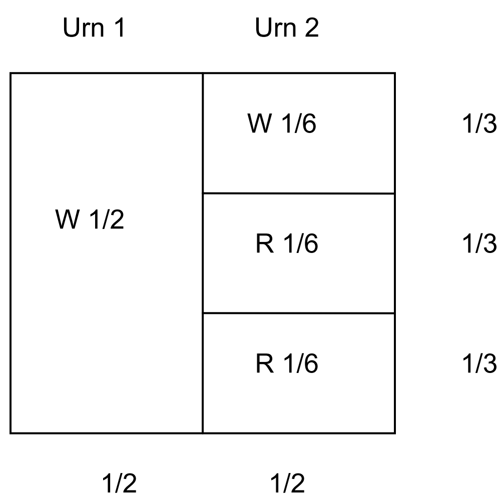
Procedure:

Steps	Questions/Math Notes
1. Read aloud the *How Much Does a Grecian Urn* (Transparency) two times. Ask participants to work in pairs to determine the best placement of the white and red marbles in the urns.	To stimulate their thinking, ask participants questions about what they are doing: *What is the sample space for this problem? Is this a finite or infinite sample space? Explain.*
2. Circulate among the groups as they work the problem. Ask participants to draw a tree diagram of each arrangement of the marbles in the two urns.	*What are all the possible outcomes?* *Have you listed all the possible ways to distribute the marbles? Justify your response.* *Is the event "draw a white marble" a simple or compound event? How do you know?* *How can you represent the probability of picking an urn?* *How can you represent the probability of picking an urn and selecting a red marble? … of picking an urn and selecting a white marble?* *Have you computed the probability of drawing white and the probability of drawing red for each case?*
3. Select several pairs to present their solution. Ask them to include a drawing that shows the distribution of the marbles in the two urns, a tree diagram that shows the possible outcomes, and the probability of winning the prize. Try to select groups that have different solutions.	*Are you sure you have found the best way to distribute the marbles? How do you know?*

Sample Space: {(Urn 1, W), (Urn 1, R), (Urn 2, W), (Urn 2, R)}

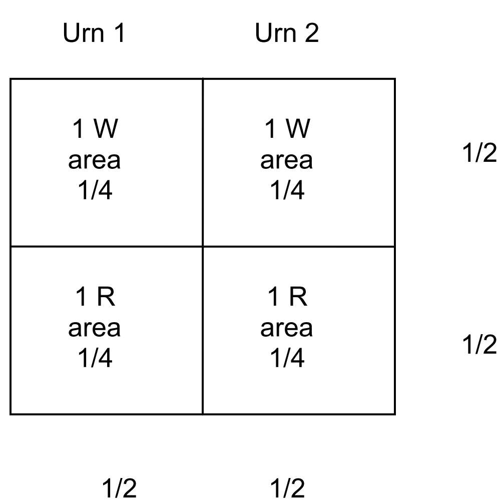
Area Models for Determining the P(W)

Case 2 example



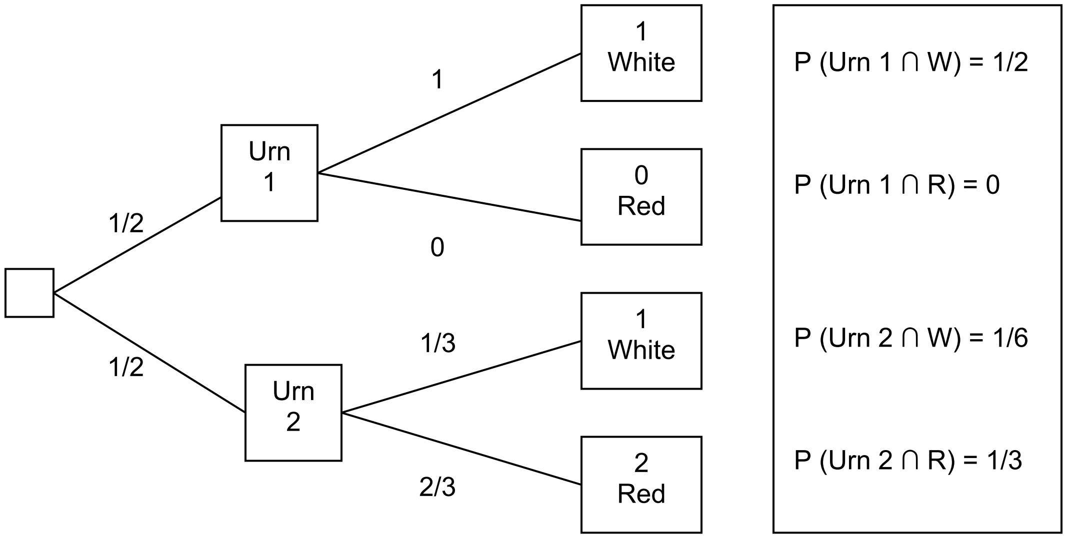
P(W) = 1/2 + 1/6 = 2/3

Case 3 example



P(W) = 1/4 + 1/4 = 1/2

Tree Diagram of One of the Best Solutions:



P (winning) = P (Urn 1 ∩ white) + P (Urn 2 ∩ white) = 1/2 + 1/6 = 3/6 + 1/6 = 4/6 or 2/3

P (losing) = P (Urn 1 ∩ red) + P (Urn 2 ∩ red) = 0 + 1/3 = 1/3

Solution Possibilities:

Case	Urn 1	Urn 2	P(white)	P(red)
Case 1: 4 marbles in Urn 1	2W, 2R	empty	1/2	1/2
Case 2: 3 marbles in Urn 1	2W, 1R	1R	1/3	2/3
Case 2: 3 marbles in Urn 1	**1W, 2R**	**1W**	**2/3**	**1/3**
Case 3: 2 marbles in Urn 1	2W	2R	1/2	1/2
Case 3: 2 marbles in Urn 1	2R	2W	1/2	1/2
Case 3: 2 marbles in Urn 1	1W, 1R	1W, 1R	1/2	1/2
Case 4: 1 marble in Urn 1	1R	2W, 1R	1/3	2/3
Case 4: 1 marble in Urn 1	**1W**	**1W, 2R**	**2/3**	**1/3**
Case 5: 0 marbles in Urn 1	empty	2W, 2R	1/2	1/2

The best solutions (shown in the table above) are the ones in bold. Both of these solutions increase the probability of earning the prize from 1/2 to 2/3.

Extension:
You and your partner have the opportunity to *urn* a prize. You are given three urns and six marbles, three red and three white marbles. You may arrange the marbles in the three urns any way you choose. Then another person will randomly draw one marble out of one of the urns. If the marble is white, you will win the prize. If the marble is red, you will win nothing. How could you best arrange the six marbles in the three urns to improve your chances of *urning* the prize?
Solution: {(Urn 1, 1W), (Urn 2, 1W), (Urn 3, 1W, 3R)}
P(*urning* the prize) = P(white) = 1/3 + 1/3 + 1/12 = 9/12 or 3/4

References:

Phillips, E., Lappan, G., Winter, M. J., & Fitzgerald, W. (1986). Activity 6: Area models. *Middle grades mathematics project: Probability* (pp. 103-106). Menlo Park, CA: Addison-Wesley.