How to Split the Pot

Purpose:
Participants will use probability to determine the fairest way to split the pot of gold when a game has to be ended before completion.

Overview:
Participants will learn of a gambling game that was played during the 1600's. The initial laws of probability were created from discussions and writings about this game. To capture the spirit of these discussions, participants (in small groups) will explore several ways to split the money that might have been wagered in this game. They will determine the fairest way to split the money and explain their rationale. Then they will learn of the correspondence about this same situation that occurred between Pascal and Fermat and how these two gentlemen decided the pot should be split.

TExES Mathematics 4-8 Competencies. The beginning teacher:

IV.013.B Uses the concepts and principles of probability to describe the outcome of simple and compound events.
IV.013.C Generates, simulates, and uses probability models to represent a situation.
IV.013.D Determines probabilities by constructing sample spaces to model situations.

TEKS Mathematics Objectives. The student is expected to:

4.13.A List all possible outcomes of a probability experiment such as tossing a coin.
4.13.B Use a pair of numbers to compare favorable outcomes to all possible outcomes.
5.12.A Use fractions to describe the results of an experiment.
6.9.A Construct sample spaces using lists, tree diagrams, and combinations.
6.9.B Find the probabilities of a simple event and its complement and describe the relationship between the two.
7.10.A Construct sample spaces for compound events (dependent and independent).
8.11.A Find the probabilities of compound events (dependent and independent). ???
8.11.B Use theoretical probabilities and experimental results to make predictions and decisions.

Terms.
Probability, deterministic vs. random experiment, sample space, event

Materials.
- 1-2 dice for each small group
- Transparency
- Activity Sheet for each participant

Transparencies.
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Activity Sheet(s).
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Procedure:

Steps	Questions/Math Notes
1. Read aloud the How to Split the Pot problem (Transparency #??) two times. Answer questions about the meaning of the problem.	Share information about the beginnings of probability (see attachment).
2. Ask participants to work in pairs or small groups to determine the best way to split the pot of money.	To stimulate their thinking, ask participants questions about what they are doing: *How many different (reasonable) ways can you split the pot of money? How do you know?* *Do you think splitting the pot 50-50 is fair? Why or why not?*
3. Circulate among the groups as they work the problem. Ask participants to draw a tree diagram which indicates the possible outcomes from each throw of the die.	*In a probability tree diagram, how can you show what happens each time a player tosses the die? How do you know?* *What probability could you assign to each possible outcome of the toss of the die? How did you come up with those values?* *What is the probability of tossing a 6 on the next throw of the die? Explain.* *What is the probability of tossing a 2 on the next throw of the die? Explain.* *What is the probability of tossing two 2's on the next two throws of the die? Explain.*
4. Select four small groups to present their solutions. Try to select groups that have different solutions.	*Is your solution more advantageous for you or for your opponent? Is your solution fair to both parties? Why or why not?* *Of the 64 pistoles bet, how many should you get and how many should your opponent get?*

Short History of the Beginning of Probability:

Probability as we know it today was launched by a trio of Frenchmen in the mid-17th Century: a high-living nobleman, the Chevalier de Méré, and two spare-time mathematicians, Blaise Pascal and Pierre de Fermat.

Pascal's major interests were philosophy and religion. Fermat was a jurist (lawyer) by profession. Pascal's mathematics interests included projective geometry; Fermat's were analytic geometry and number theory.

In 1651 De Méré and Pascal found themselves together on a trip and began searching for a mutually interesting topic of conversation to lighten their journey. The worldly De Méré presented the spiritual Pascal with a mathematical problem which had fascinated sporting bloods since the Middle Ages: how to split the pot in a dice game that has to be discontinued. Pascal pondered the problem for a couple of years and in 1654 relayed it to Fermat.

In the celebrated correspondence which ensued over De Méré's poser, Pascal and Fermat began by agreeing that in a discontinued dice game the stakes on the table should be divided according to the prospects each player has of winning.

Insert problem here. Suppose that De Méré and one of his cronies were actually playing the dice game of those times. Each player had bet 32 pistoles that his chosen number will turn up three times on a die before the other player's number has done so. After the game has been under way for a while, De Méré's number, 6, has turned up twice; his opponent's 2 has turned up only once. At this point De Méré receives a sudden summons to an audience with the young King Louis XIV. How should the players split the 64 pistoles on the table?

Solution:

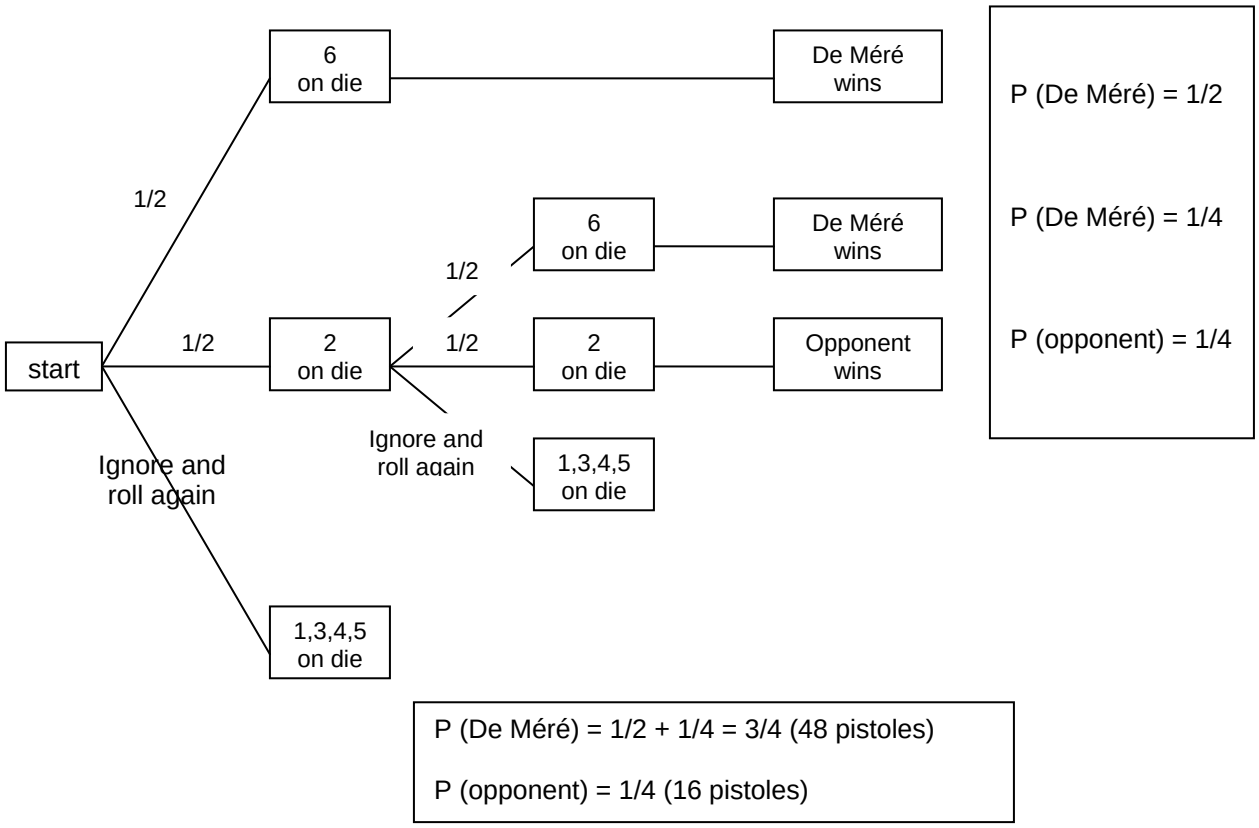
De Méré's opponent could challenge that each number is equally likely to appear on any given toss of the die so the pot should be split 50-50.

If the 50-50 split is rejected, De Méré's opponent could contend that since his chances of getting two lucky throws are half as good as De Méré's chances of getting one lucky throw, he is entitled to half as much of the pot as De Méré: 21 1/3 pistoles to De Méré's 42 2/3. In other words, the opponent gets 1/3 of the pot and De Méré gets 2/3 of the pot of money.

De Méré, on the other hand, could contend that on the next throw of the die the worst that could happen to him would be to lose his advantage, in which case the game would be even and he would be entitled to an even-steven cut of 32 pistoles. If, however, his next throw was lucky, he would win the original bet and pick up all 64 pistoles. De Méré argues, therefore, that even before the throw he is entitled to the 32 pistoles he is sure of, plus 16 more he is half-sure of.

Pascal and Fermat agreed with De Méré. Out of the letters Pascal and Fermat wrote to one another on this topic and other various gambling situations evolved the laws of chance. The idea that chance is governed by laws may seem unconvincing to anyone persuaded of the rule of Lady Luck. But actually the laws of chance do not preclude the possibility that an individual will enjoy a stroke of luck. Nor do they deny the value of playing hunches. They begin to act as laws only when many instances are involved—many throws of the dice, many deals of the cards, many car collisions, many lifetimes. This aspect of probability is known as the *law of large numbers.*

Tree Diagram:
De Méré's chosen number is 6 and his opponent's is 2.
De Méré already has two 6's and his opponent has one 2 when they are called for an audience with the king and the game must end. How should they split the pot of money?

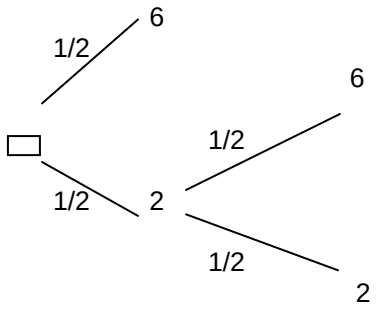


P (De Méré) = 1/2 + 1/4 = 3/4 (48 pistoles)

P (opponent) = 1/4 (16 pistoles)

In essence (with the rolls of 1, 3, 4, and 5 being ignored), the tree diagram looks very similar to the one we created for the Garbage In, Garbage Out Activity.

Refined Tree Diagram:



P(6) = 1/2 De Méré Wins

P(2, 6) = 1/4 De Méré Wins

P(2,2) = 1/4 Opponent Wins

References:
Bergamini, D. (1963). Figuring the odds in an uncertain world. In R. Dubos, H. Margenau, & C. P. Snow (Eds.), *Life science library: Mathematics* (pp. 126-129). New York: Time Inc.