Garbage In, Garbage Out!

Teacher Notes

Purpose:
Participants will conduct an experiment to determine the probability of having to take the garbage out by playing a game with given conditions.

Overview:
In small groups, participants will investigate whether the tossing of a coin given certain conditions is a fair game. They will conduct an experiment to simulate the problem situation and examine the results to determine experimental probabilities.

TExES Mathematics 4-8 Competencies. The beginning teacher:

IV.013.A Explores concepts of probability through data collection, experiments, and simulations.

IV.013.D Determines probabilities by constructing sample spaces to model situations.

TEKS Mathematics Objectives. The student is expected to:

4.13.A List all possible outcomes of a probability experiment such as tossing a coin.

5.12.A Use fractions to describe the results of an experiment.

5.12.B Use experimental results to make predictions.

6.9.A Construct sample spaces using lists, tree diagrams, and combinations.

6.9.B Find the probabilities of a simple event and its complement…

8.11.B Use theoretical probabilities and experimental results to make predictions and decisions.

Terms.
Probability, fair coin, outcomes, equally likely outcomes, sample space, deterministic vs. random experiment, tree diagram

Materials.

- One penny for each pair of participants
- Activity sheet and Transparency

Transparency:

- *Garbage In, Garbage Out!*

Activity Sheet:

- *Garbage In, Garbage Out!*

Procedure:

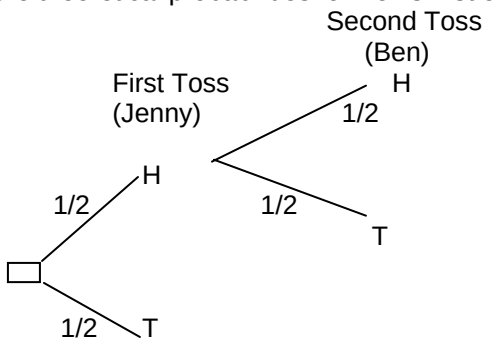
Steps	Questions/Math Notes
1. Have participants read the problem "*Garbage In, Garbage Out*" (Transparency) and restate it in their own words. Have them work in pairs to conduct an experiment that simulates the conditions stated in the problem. Ask them to record their results in a table for discussion.	To stimulate their thinking, ask participants questions about how they are thinking about the problem. *Is this a deterministic or random experiment? Explain.* *What model will you use for this experiment? Describe at least two other models that could be used?* *If you use two coins, what assumptions are you making about these coins?* *What does the toss of the first coin represent? What does the toss of the second coin represent?* *Describe a trial for this situation. What does a trial represent?* *How many trials will you conduct and why?* *What are the possible outcomes in this game? Are the outcomes equally likely? Explain.* *Describe the desired outcome for Jenny to win ; for Ben to win.*
2. Monitor the groups as they conduct the experiment. Ask participants to record their results of the experiment in a table to share with the whole group when finished. Have them use their results to answer the question in the problem and put their work on transparencies.	*How many trials have you conducted? What do the results of these trials tell you about the probabilities?* *What do you think will happen if you were to conduct more trials? Do you think that your experimental probabilities will change? Explain.* *Conduct more trials until you are confident about the probabilities from your experiment.* *Use your results to answer the question "Is this game fair?"*
3. Have several groups share their work with the whole group. Ask them to discuss the probability of Jenny and Ben winning based upon their experimental results. Select groups with varied solution strategies for the discussions.	*How did you determine if the game was fair or not?* *If the game was not fair, whom did the game favor and why?* *How could you model the results of this experiment without using simulation by tossing the coins?* *How do you think the probabilities from your model would compare with your experimental results?*

Procedure:

Steps	Questions/Math Notes
4. Combine the data from the different groups.	Ask participants to reflect upon the data collected from all groups. *How do your experimental results compare with the other groups' results?* *What information can you get from your experimental results? From the whole groups' experimental results?*
5. Discuss the "Law of Large Numbers".	*How will combining the data from all the groups (increasing the sample size) affect the probabilities in this problem?*
6. Ask participants to compute the theoretical probabilities for the original problem.	*How did you compute the theoretical probabilities for this problem?* *How do your empirical/experimental results compare with the theoretical probabilities?*
7. Have participants graph their own data and compare the graph to that of y= theoretical probability. Have them link their calculators with other groups so that they have increased their sample size and graph this data.	Ask participants to state their observations based upon these graphs. *What does the graph of the line y=theoretical probability represent?* *How does your data relate to this line?* *What do you observe about the shape of the data from the larger sample size?*

Solution: Possible outcomes for Sample Space: **{(HH), (HT), (T)}**
Favorable outcomes for Jenny to win: {(HH)}
Favorable outcomes for Ben to win: {(HT), (T)}

Participants will record the results of the tossing of the coins on tables (Activity Sheet #) . Their experimental results should indicate that the game is not fair. A tree diagram can be used to find the theoretical probabilities for Ben's first offer as shown below.



P(HH) = 1/4 Jenny Wins

P(HT) = 1/4 Ben Wins

P(T) = 1/2 Ben Wins

P(Ben Wins) = 3/4 and P(Jenny Wins) = 1/4

Ben's second offer is also not fair since P(Ben Wins) • 1 ≠ P(Jenny Wins) • 2 .

$3/4 \bullet 1 \neq 1/4 \bullet 2$.

$3/4 \neq 1/2$

Extension:
How could you make this game fair given the conditions in the first paragraph?

Possible Solution:
Ben could take out the garbage three times each time Jenny wins.

Reference:
Shulte, A., & Choate, S. (1977). Activity 1: Ginny's Game. *What Are My Chances? Book B* (p. 21). Mountain View, CA: Creative Publications.