A Game of Darts, Vader?

Purpose:
Participants will investigate probabilities using area models and make connections to multiplication using a dart game with a Tangram dart board.

Overview:
Participants will work in small groups to investigate the probability of landing on certain shapes on a Tangram dart board. They will determine if the game presented in the problem is fair by using the ratio of two areas to represent probability.

TexES Mathematics 4-8 Competencies. The beginning teacher:

IV.013.B Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
IV.013.C Generates, simulates, and uses probability models to represent a situation.
IV.013.D Determines probabilities by constructing sample spaces to model situations.
IV.013.E Solves a variety of probability problems using combinations, permutations, and geometric probability (i.e. probability as the ratio of two areas).

TEKS Mathematics Objectives. The student is expected to:

4.12 Measure to solve problems involving length, including perimeter, time, temperature, and area.
5.7B Use critical attributes to define geometric shapes or solids.
5.11A Measure to solve problems involving length (including perimeter)... , and area.
6.8B Select and use appropriate units, tools, or formulas to solve problems involving length (including perimeter and circumference, area...
6.9A Construct sample space using lists, tree diagrams, and combinations; and
6.9B Find probabilities of a simple event and its complement and describe the relationship between the two.
7.6B Use properties to classify shapes including triangles, quadrilaterals, pentagons, and circles.
7.9 Estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

Terms.
Probability, ratio, parallelogram, right triangle, square, area, Tangram

Materials.
- 1 set of Tangrams per group of four (optional)

Transparencies.
- *A Game of Darts, Vader?*

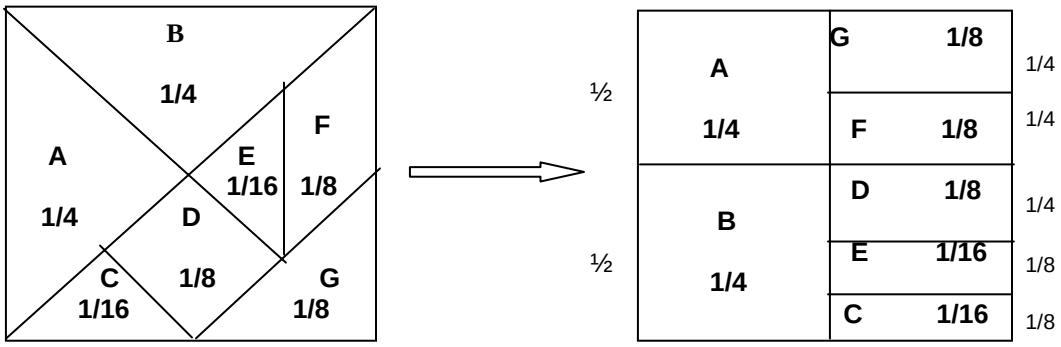
Activity Sheet(s).
- *A Game of Darts, Vader?*

Procedure:

Steps	Questions/Math Notes
1. Place Transparency on the overhead and have participants read the problem. Have them work individually on the problem for about 5 minutes and then share solution strategies within a group of four.	As participants work within their groups, ask them questions to assess their understanding of the problem. *How do the areas of the different geometric figures relate?* *How does the probability of a dart landing on a given geometric figure relate to the area of that figure?* *What must be true for the game to be fair?*
2. Have the groups develop a solution strategy and write it on a transparency.	*How did you use the area of each Tangram shape to help you determine if the game was fair?* *How could you represent these Tangram areas another way? Explain.* *How can you use area to assign point(s) to the different Tangram shapes on the dart board to make the game fair?*
3. Select groups with different solution strategies to share with the whole group. Ask summarizing questions to debrief the activity.	*What does P(A) = 1/4 mean in this problem ?* *Suppose the dimensions of the square are changed so that each side is 2 feet. How would this change affect the probabilities?* *How would any change in the dimensions of the given square affect the probabilities?* *Suppose the dart board is in the shape of a circle. How could you determine the degrees of each sector so that probabilities of the original problem remain the same?* *Describe other geometric figures that could be used as a model for the dart board in the original problem.* *How could you generalize a rule for finding the probability of a geometric region?* *What is the sum of the probabilities of a dart landing in each region on the Tangram dart board?*

Solution: The game is not fair because the product of the probability of landing on any geometric figure times the assigned point value is not the same for all regions.

Note: A set of Tangrams can be used to determine the area of each region.



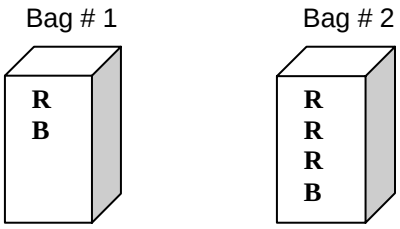
P(A) = (1/2) (1/2) = 1/4
P(B) = (1/2) (1/2) = 1/4
P(C) = (1/2) (1/8) = 1/16
P(D) = (1/2) (1/4) = 1/8
P(E) = (1/2) (1/8) = 1/16
P(F) = (1/2) (1/4) = 1/8
P(G) = (1/2) (1/4) = 1/8

To make the game fair, sub-divide each rectangular region in the square on the right into areas of 1/16 square foot and assign points so that each original region has the same weighted point value.

A	**G**
A	**G**
A	**F**
A	**F**
B	**D**
B	**D**
B	**E**
B	**C**

If the dart lands on regions A or B, a score of 1 point can be assigned; if it lands on regions D,F, or G, then 2 points can be scored. Landing on regions E or C can be assigned 4 points.

Extension:
4 red and 2 blue color tiles are placed into 2 bags as shown below. Use an area model to find the probability of drawing a red color tile by reaching into a bag?

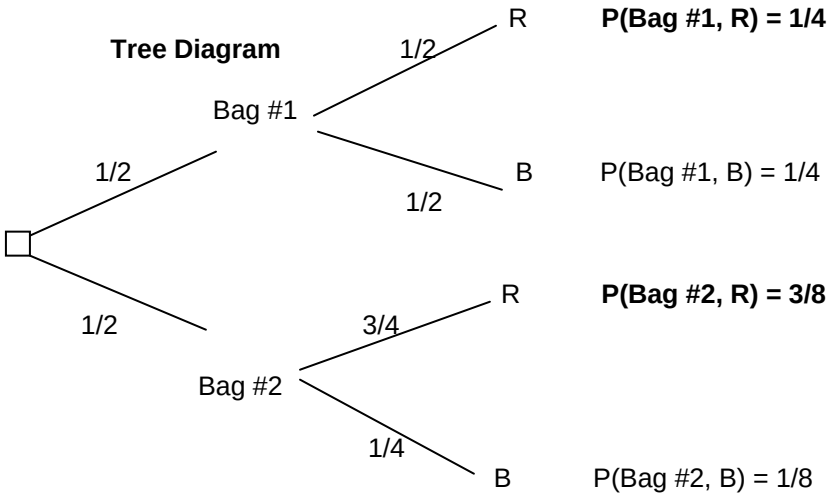


Validate your solution using a tree diagram and make connections among the different representations.

Solution: P(R) = P(Bag #1,R) + P(Bag #2, R) =1/4 + 3/8 = 5/8

Area Model : 1 square unit

Bag #1	**Bag #2**
R **1/4**	**R** **1/8** **R** **1/8**
B **1/4**	**R** **1/8** **B** **1/8**



Using a tree diagram, P(Bag #1, R) + P(Bag #2,R) = 1/4 + 3/8 = 5/8. The area model provides a visual representation of the products in the tree diagram. For example, P(Bag #1,R) = (1/2) • (1/2) or 1/4 in the area model.