With and Without Ewe

Purpose:
Participants will determine probabilities of seeing a black Sleepy Sheep under replacement and non-replacement conditions.

Overview:
In small groups, participants will determine the probability of seeing a black Sleepy Sheep under replacement and non-replacement conditions. They will have the opportunity of using a problem space model to represent the emerging probabilities, adding to their representational abilities.

TExES Mathematics Competencies. The beginning teacher:

IV.13.C	Generates, simulates, and uses probability models to represent a situation.
IV.13.D	Determines probabilities by constructing sample spaces to model situations.
IV.14.B	Demonstrates an understanding of random samples, sample statistics, and the relationship between sample size and confidence intervals.

TEKS Mathematics Objectives. The student is expected to:

4.13.A	List all possible outcomes of a probability experiment such as tossing a coin.
6.10.A	Draw and compare different graphical representations of the same data.
6.10.D	Solve problems by collecting, organizing, displaying, and interpreting data.
7.11	Understand that the way a set of data is displayed influences its interpretation.
7.11.A	select and use an appropriate representation for presenting collected data and justify the selection.
7.4.A	Generate formulas involving conversions, perimeter, area, circumference, volume, and scaling.
8.11	Apply concepts of theoretical and experimental probability to make predictions.
8.11.B	use theoretical probabilities and experimental results to make predictions and decisions.
8.11.C	Select and use different models to simulate an event.

Terms.
Replacement, sample space, probability, random sample, probability experiment

Materials.
For each small group of participants:

- Transparency
- Activity Sheet for each participant
- Graph paper

Transparencies.

- Aunt Sarah and the Farm

Activity Sheet(s).

- Aunt Sarah and the Farm

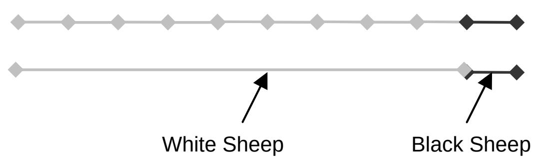
Procedure:

Steps	Questions/Math Notes
1. Read aloud the With and Without Ewe (Transparency #??) two times. Allow participants to ask questions about the problem situation described. Ask participants to work in groups of 4 to begin exploring some of the mathematics of the situation.	To stimulate reflection, remind participants that the case where the sleeper needs only count to ten is **without replacement**, when the sleeper must count higher is **with replacement**. For the case involving replacement ask: *Is there a way to predict what happens in the 5^{th} jump? The 6^{th}? Any jump?* *How can we organize our work to make prediction easier to visualize?* (Demonstrate the Probability Space Diagrams).
2. Illustrate how to use Probability Space Diagrams to represent various probability experiments. *(See Discussion section which follows)*.	
3. Circulate among the groups as they work the problem. Encourage each group to explore each scenario and the impact upon the sample space contained therein. Ask participants to graphically represent each scenario using a Problem Space Diagram.	*Remind participants that the question requires each jump to be considered.* *For the first case this means that the sample space decreases by one with each jump.* *(Geometric distribution?)* *This actually leads to a situation where the with replacement section is a simpler model, which is not usually the case!*
4. Select several small groups to present their findings and graphics.	*How does the Probability Space Diagram reflect the problem situation?*

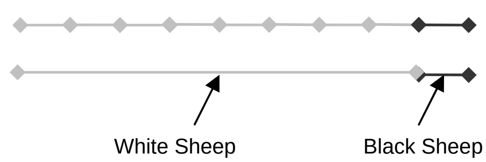
Probability Space Diagrams

Probability Space Diagrams were invented by Peter Cheng to test ideas about the fundamental role of representational systems in conceptual learning if probability. Since their inception research has shown that using Problem Space Diagrams enables learners to develop a deeper understanding of this difficult topic. The **With and Without Ewe** activity provides an ideal situation to explore how these diagrams can be used to represent the underlying mathematical concepts.

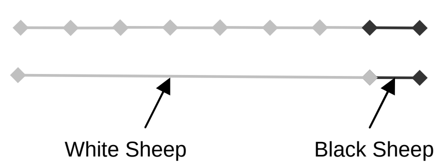
Of the two scenarios, the first is in many ways the more interesting. Since the sheep will not return to the jumping pool the sample space decreases by one with each jump of the sheep. Using a Probability Space Diagram we can show the sequence of jumps as follows:



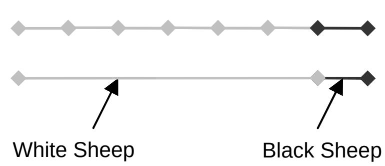
DAY ONE:
Since there are ten separate sheep and only one of them is black we see that relative to the full set of possibilities there is one chance in ten of seeing the black sheep on the first jump. P= .10



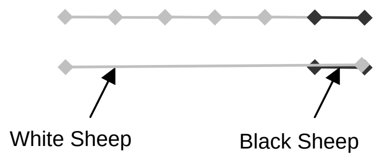
DAY TWO:
Assuming we did not see the black sheep on the previous jump there are now nine separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in nine of seeing the black sheep on the first jump. P= $.\overline{11}$



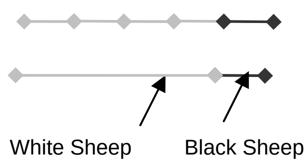
DAY THREE:
Assuming we did not see the black sheep on the previous jump there are now eight separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in eight of seeing the black sheep on the first jump. P= .125



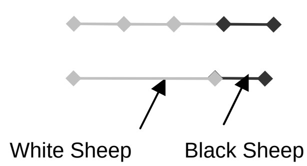
DAY FOUR:
Assuming we did not see the black sheep on the previous jump there are now seven separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in seven of seeing the black sheep on the first jump. P= $.\overline{148257}$



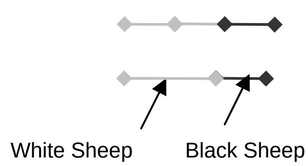
DAY FIVE:
Assuming we did not see the black sheep on the previous jump there are now six separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in six of seeing the black sheep on the first jump. P= .166



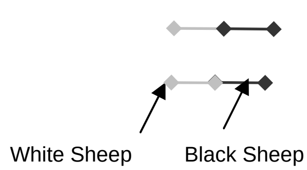
DAY SIX:
Assuming we did not see the black sheep on the previous jump there are now five separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in five of seeing the black sheep on the first jump. P= .20



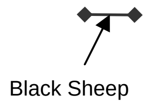
DAY SEVEN:
Assuming we did not see the black sheep on the previous jump there are now four separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in four of seeing the black sheep on the first jump. P= .25



DAY EIGHT:
Assuming we did not see the black sheep on the previous jump there are now three separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in three of seeing the black sheep on the first jump. P= .33

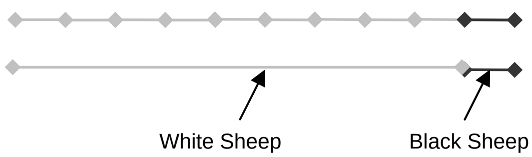


DAY NINE:
Assuming we did not see the black sheep on the previous jump there are now six separate sheep and only one of them is black. Relative to the full set of possibilities there is now one chance in six of seeing the black sheep on the first jump. P= .5



DAY TEN:
Assuming we did not see the black sheep on the previous jump there is only the black sheep left.
P= 1

Of the remaining scenario, the second is much easier to describe. Since the sheep will always return to the jumping pool the sample space will remain constant for each jump. Using a Probability Space Diagram each jump would be as follows:



DAY N:
Since there are always ten sheep and only one of them is black we see that relative to the full set of possibilities there is one chance in ten of seeing the black sheep on any jump.
P= .10

References:

http://www.cogs.susx.ac.uk/lab/reps/LEDs/probability/index.html

Cheng, P. C.-H., & Pitt, N. G. (2003). Diagrams for difficult problems in probability. Mathematical Gazette, 87(508), 86-97.

Cheng, P. C. H. (2003). Diagrammatic re-codification of probability theory: A representational epistemological study. In Proceedings of the Twenty Fifth Annual Conference of the Cognitive Science Society. Mahwah, NJ: Lawrence Erbaum.