Does One-and-One Make Two?

Purpose:
Participants will determine the probability of simple events.

Overview:
In small groups, participants will explore both experimentally and theoretically the probability of making 0, 1, and 2 free throw shots in a basketball one-and-one situation. Participants will first create a simulation of the free throw situation to determine the probabilities of making 0, 1, and 2 free throws; then they will draw a tree diagram and determine the theoretical probabilities of each event.

TExES Mathematics 4-8 Competencies. The beginning teacher:

IV.013.A Explores concepts of probability through data collection, experiments, and simulations.
IV.013.B Uses the concepts and principles of probability to describe the outcome of simple and compound events.
IV.013.C Generates, simulates, and uses probability models to represent a situation.
IV.013.D Determines probabilities by constructing sample spaces to model situations.

TEKS Mathematics Objectives. The student is expected to:

4.13.A List all possible outcomes of a probability experiment such as tossing a coin.
4.13.B Use a pair of numbers to compare favorable outcomes to all possible outcomes.
5.12.A Use fractions to describe the results of an experiment.
5.12.B Use experimental results to make predictions.
6.9.A Construct sample spaces using lists, tree diagrams, and combinations.
6.9.B Find the probabilities of a simple event and its complement and describe the relationship between the two.
7.10.A Construct sample spaces for compound events (dependent and independent).
7.10.B Find the approximate probability of a compound event through experimentation.
7.11.B Make inferences and convincing arguments based on an analysis of given or collected data.
8.11.A Find the probabilities of compound events (dependent and independent).
8.11.B Use theoretical probabilities and experimental results to make predictions and decisions.
8.11.C Select and use different models to simulate an event.

Terms.
Probability, sample space, event, simulation, tree diagram

Materials.
For each small group of participants: two dice, two coins, one deck of cards, two spinners, one set of number tiles 0-9, 5 marbles: 2 red and 3 yellow, activity sheet

Transparencies.

- *Does One-and-One Make Two?*

Activity Sheet(s).

- *Does One-and-One Make Two?*

Procedure:

Steps	Questions/Math Notes
1. Read aloud the *Does One-and-One Make Two?* (Transparency) two times. Ask participants to work in small groups to determine whether Michele is most likely to score 0, 1, or 2 points.	To stimulate their thinking, ask participants questions about what they are doing: *What is the sample space for this problem? Is this a finite or infinite sample space? Explain.*
2. Ask participants to choose one of models to use to simulate the problem. Don't intervene too quickly. Give the small groups time to experiment and determine whether their model works or doesn't work. You can hinder a wonderful learning opportunity if you interfere too soon. Once the group has chosen a reasonable model, ask them to run 50 trials of the experiment.	*Can the two coins simulate the experiment?* *What will one flip of a coin represent?* (No, two coins do not provide a good simulation model. We are tempted to say that one flip of a coin can represent a free throw, but this would work only if Michele were a 50% free-throw shooter. Students are tempted to tape 60% on one side of the coin and 40% on the other side of the coin. This doesn't work, however, because the flip of the coin remains 50-50, not 60-40.) *Can the two dice simulate the experiment?* *What will one roll of a die represent?* (No, two dice do not provide a good simulation model. We are tempted to say that one roll of a die can represent the first free throw, but there is no way to represent the 60%. Each side of the die represents a 16 2/3% chance.) *Can a deck of cards be used to simulate the experiment? What will one draw from the deck represent?* (Yes, we could use ten cards from the deck to simulate this problem and allow 4 cards (40%) to represent a miss and 6 cards (60%) to represent a hit. For example, we might choose the Ace through 10 of Clubs. A draw of the Ace, 2, 3, or 4 will represent a missed free throw. A draw of a 5, 6, 7, 8, 9, or 10 will represent a good free throw. We will draw once. If the card is Ace-4, we score 0 points. If the card is 5-10, we return the card to the deck of 10 cards and draw again to determine whether we scored 1 or 2 points.) *Can two spinners be used to simulate the experiment?* (A spinner can be used but only if it has divisions that allow for a 60-40 split. For example, if the spinner is divided into ten equal sections. Then four of the sections could

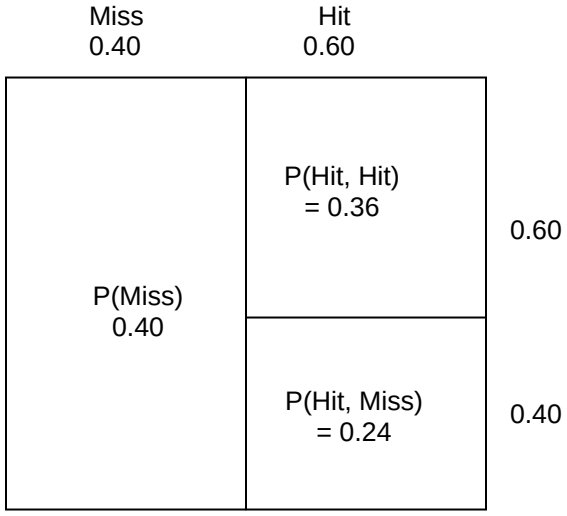
	represent a "miss" and six of the sections could represent a "hit".) *Can number tiles 0-9 be used to simulate the experiment?* (Yes, four numbers would represent a "miss" and six numbers would represent a "hit". We would draw one number from the bag. If we got a hit, we would replace the number and draw again. A "miss" = 0 points. A "hit followed by a miss" = 1 point. A "hit followed by a hit" = 2 points.) *Can the 5 marbles be used to simulate the experiment?* (Yes, the two red marbles (40% of total) would represent a "miss" and the three yellow marbles (60% of total) would represent a "hit". We would draw one marble from a bag, replace the marble, and draw a second marble provided the first marble was yellow. A draw of a red marble would indicate 0 points were scored. A draw of a yellow marble followed by a red marble would indicate 1 point was scored. A draw of a yellow marble followed by a yellow marble would indicate 2 points were scored.)
3. Select several small groups to present the model they used. Ask them to describe how the model worked and to summarize their findings. Try to select groups that used different models.	*How did you use your model to represent the two possible free-throw shots?* *How did your model differentiate between the 60% chance of making the shot and the 40% chance of missing the shot?* *Show how you used your model to simulate the experiment. How did you model Michele's first shot? If Michele makes the first shot, then how did you model Michele's second shot?*
4. Ask participants to draw an area model or a tree diagram of the one-and-one free-throw situation and determine whether Michele is most likely to get 0, 1, or 2 points.	*Have you listed all the possible ways for Michele to score points? Justify your response.* *Is the event "score 2 points" a simple or compound event? How do you know?* *How can you represent the probability of scoring 2 points at the free-throw line during a one-and-one situation?*
5. Select 1-2 small groups to present their solution. Ask them to include a tree diagram that shows the possible	*Do the theoretical probabilities match your experimental probabilities? Why or why not?*

outcomes, and the probabilities of each event.	*Do the theoretical probabilities seem reasonable?* *How do you know you have accurately computed the theoretical probabilities of scoring 0, 1, and 2 points?*

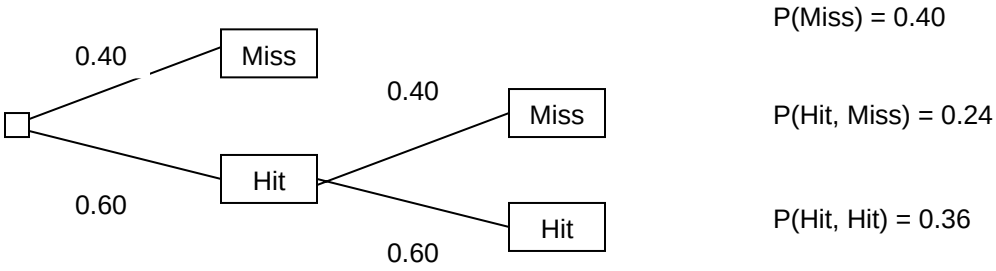
Sample Space: {(miss), (hit, miss), (hit, hit)}

Area Model

Error!



Tree Diagram:



Solution:
Michele is most likely to score 0 points (probability 0.40) when she goes to the free-throw line during a one-and-one situation. She is least likely to score 1 point (probability 0.24).

Extension:
How does the problem change if Michele is 65% free-throw shooter?

P(Miss) = 0.35
P(Hit, Miss) = 0.65 x 0.35 = 0.2275
P(Hit, Hit) = 0.65 x 0.65 = 0.4225

Michele is most likely to score 2 points. She is least likely to score 0 points.

References:
Phillips, E., Lappan, G., Winter, M. J., & Fitzgerald, W. (1986). Activity 7: Expected value. *Middle grades mathematics project: Probability* (pp. 115-128). Menlo Park, CA: Addison-Wesley.