Game of Chuck-a-Luck

Purpose:
Participants will determine the expected outcome of playing the game of Chuck-a-Luck.

Overview:
In small groups, participants will investigate the possible financial outcome of playing the game of Chuck-a-Luck over time. They will decide whether the game is fair.

TExES Mathematics 4-8 Competencies. The beginning teacher:

IV.013.A	Explores concepts of probability through data collection, experiments, and simulations.
IV.013.B	Uses the concepts and principles of probability to describe the outcome of simple and compound events.
IV.013.D	Determines probabilities by constructing sample spaces to model situations.
IV.013.F	Uses the binomial, geometric, and normal distributions to solve problems.

TEKS Mathematics Objectives. The student is expected to:

4.13.A	List all possible outcomes of a probability experiment such as tossing a coin.
5.12.A	Use fractions to describe the results of an experiment.
5.12.B	Use experimental results to make predictions.
6.9.A	Construct sample spaces using lists, tree diagrams, and combinations.
6.9.B	Find the probabilities of a simple event and its complement and describe the relationship between the two.
7.10.A	Construct sample spaces for compound events (dependent and independent). ???
7.11.B	Make inferences and convincing arguments based on an analysis of given or collected data.
8.11.A	Find the probabilities of compound events (dependent and independent). ???
8.11.B	Use theoretical probabilities and experimental results to make predictions and decisions.

Terms.
Probability, odds, random variable, sample space, expected value, weighted arithmetic mean

Materials.
For each small group of participants:

- 3 dice for each small group
- Transparency
- Activity Sheet for each participant

Transparencies.

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Activity Sheet(s).

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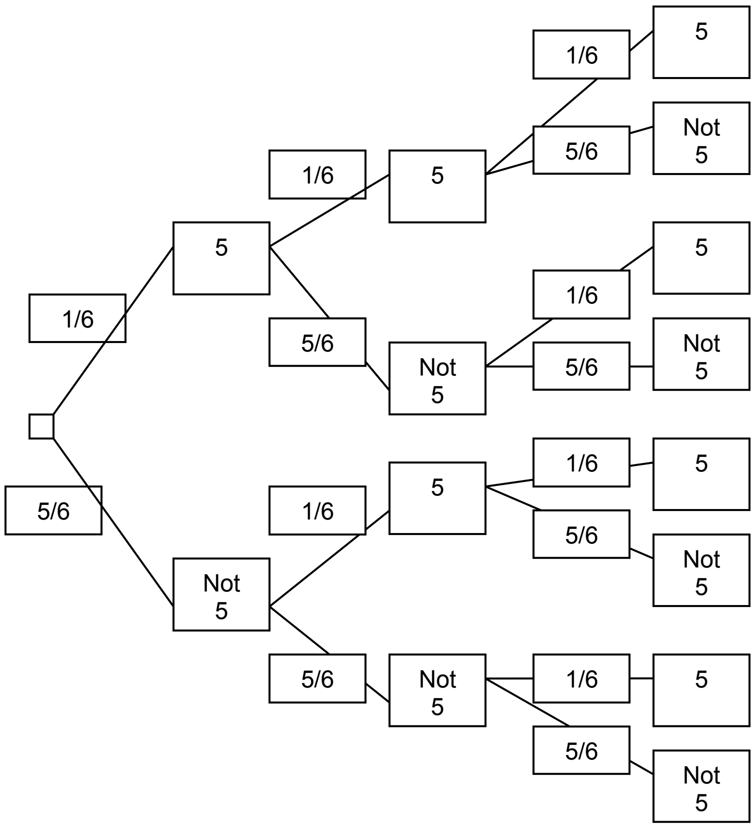
Procedure:

Steps	Questions/Math Notes
1. Read aloud the Game of Chuck-a-Luck (Transparency #??) two times. Allow participants to ask questions about the rules and play of the game. Ask participants to work in pairs to determine the amount of money they would expect to earn or lose (expected value) from playing this game.	To stimulate their thinking, ask participants questions about what they are doing: *What is the sample space for this problem? Is this a finite or infinite sample space? Explain.* *Does the $1 charge to play the game seem reasonable?*
2. Circulate among the groups as they work the problem. Ask participants to draw a tree diagram of each arrangement of the marbles in the two urns.	*What are your chances of rolling at least one 5? Do your chances improve when you are rolling three dice? Why?* *Is the event "roll at least one 5" a simple or compound event? How do you know?* *What are the odds of rolling one 5 on one die? What are the odds of rolling one 5 on three dice? What are the odds of rolling two 5's on three dice? What are the odds of rolling three 5's on three dice? How do you know?* *What will the tree diagram that represents play of the Chuck-a-Luck Game look like?*
3. Select several small groups to present their solution. Ask them to include a tree diagram that shows the possible outcomes, and the probability of each outcome occurring. Also ask them to show the expected value of one play of the game. Try to select groups that have different solutions.	*Is the Chuck-a-Luck Game fair? How do you know?* *Would you expect to win or lose if you placed one bet? How much would you expect to win or lose if you placed approximately 100 bets?* *How could you make the game fair?*

Sample Space: Let random variable X = number of 5's showing on the 3 dice. X = $\{0, 1, 2, 3\}$

Tree Diagram of Chuck-a-Luck Game:



P(X=3) = $(1/6)^3\ (5/6)^0$

P(X=2) = $(1/6)^2\ (5/6)^1$

P(X=2) = $(1/6)^2\ (5/6)^1$

P(X=1) = $(1/6)^1\ (5/6)^2$

P(X=2) = $(1/6)^2\ (5/6)^1$

P(X=1) = $(1/6)^1\ (5/6)^2$

P(X=1) = $(1/6)^1\ (5/6)^2$

P(X=0) = $(1/6)^0\ (5/6)^3$

P(X=3) = 1/216 P(X=2) = 15/216 P(X=1) = 75/216 P(X=0) = 125/216

E(X) = -\$1 (125/216) + \$1 (75/216) + \$2 (15/216) + \$3 (1/126) = -\$(17/216) ≈ -\$0.08

On average, you should expect to lose approximately \$0.08 each time you play a round of the game. In approximately 100 games, one should expect to lose \$8.

Although the first statement does not make much sense, it does provide us with information that allows us to make predictions about what will happen in the long run.

Connection to Binomial Distributions and Pascal's Triangle:
Let random variable X = number of 5's showing on three dice.

For a fixed value of *n* independent trials, the probability distribution of X is:

x	0	1	2	3
f(x)	125/216	75/216	15/216	1/216
Number of paths from tree diagram	1	3	3	1
Number of paths from Pascal's triangle	C(3,0)	C(3,1)	C(3,2)	C(3,3)

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Pascal's Triangle connected to number of combinations

C(0,0)
C(1,0) C(1,1)
C(2,0) C(2,1) C(2,2)
C(3,0) C(3,1) C(3,2) C(3,3)
C(4,0) C(4,1) C(4,2) C(4,3) C(4,4)
C(5,0) C(5,1) C(5,2) C(5,3) C(5,4) C(5,5)
C(6,0) C(6,1) C(6,2) C(6,3) C(6,4) C(6,5) C(6,6)

In general, $P(X = x) = C(n,x) \cdot p^{x} \cdot q^{(n-x)}$
For example, $P(X = 2) = C(3,2) \cdot (1/6)^{2} \cdot (5/6)^{1} = 3 \cdot (1/36) \cdot (5/6) = 15/216$

Extension:
How could you make the Chuck-a-Luck Game fair?

One Possible Solution: Let all the rules of the game stay the same except you are paid $3 if two 5's occur and $5 if three 5's occur. Then the expected value of one play of the game is:
E(X) = -$1 (125/216) + $1 (75/216) + $3 (15/216) + $5 (1/216) = $0

References:
Blakeslee, D. W., & Chinn, W. G. (1975). The expected value of a random variable. *Introductory statistics and probability* (pp. 128-131). Boston, MA: Houghton Mifflin.