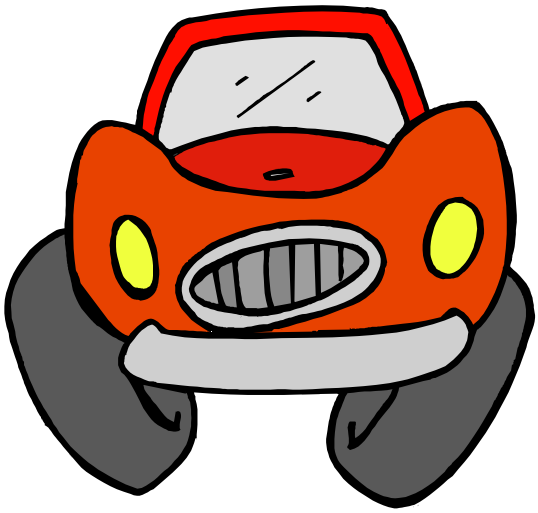
Car Trouble



The local garage employs two mechanics, Axle and Sparky, but you never know which mechanic will be working on your car. Your neighborhood consumer club has found that Axle does twice as many jobs as Sparky, Axle does a good job three out of four times, and Sparky does a good job only two out of five times. If you plan to take your car in for repairs,

a. **what is the probability that a good job will be done?**

b. **what is the probability that Sparky worked on your car if a good job is done?**

c. **what is the probability that a good job is done if Axle worked on your car?**

Car Trouble

Sample Space:

{(Axle, good job), (Axle, bad job), (Sparky, good job), (Sparky, bad job)}

Area Model:

Axle 2/3	**Sparky 1/3**
P(Axle ∩ good job) = 2/3 x 3/4 = 1/2	**P(Sparky, good job) = 1/3 x 2/5 = 2/15**
P(Axle ∩ bad job) = 2/3 x 1/4 = 1/6	**P(Sparky ∩ bad job) = 1/3 x 3/5 = 1/5**

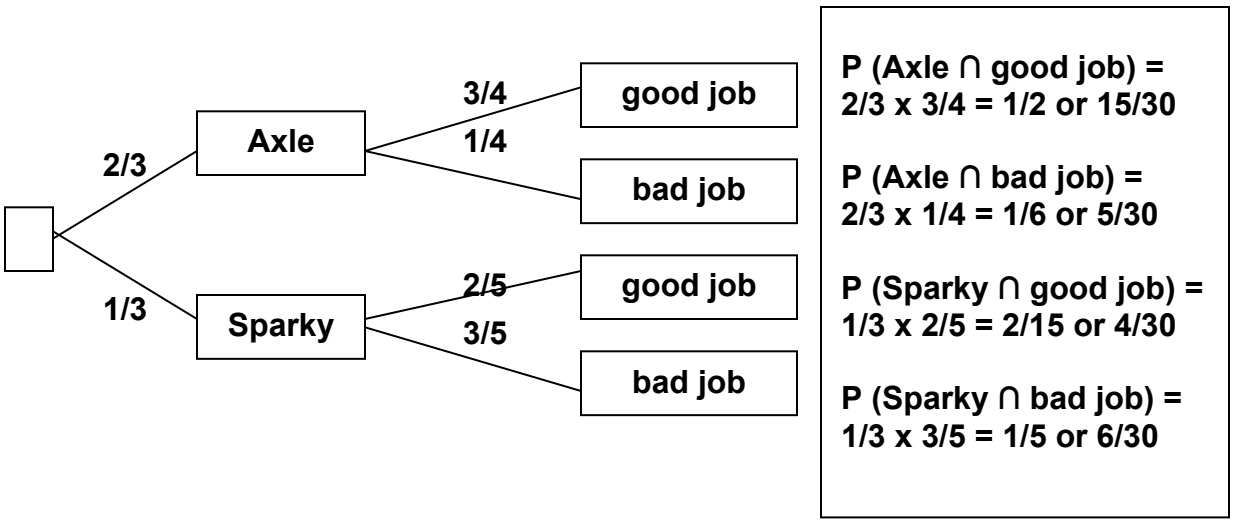
P(Axle ∩ good job) = 1/2 or 15/30

P(Axle ∩ bad job) = 1/6 or 5/30

P(Sparky ∩ good job) = 2/15 or 4/30

P(Sparky ∩ bad job) = 1/5 or 6/30

Tree Diagram:



Solution:

Use the formula for conditional probability which is
$P(A \mid B) = P(A \cap B) \div P(B)$

a. $P(\text{good job}) = P(\text{Axle} \cap \text{good job}) + P(\text{Sparky} \cap \text{good job}) =$
$15/30 + 4/30 = 19/30$

b. Use the formula for conditional probability which is
$P(A \mid B) = P(A \cap B) \div P(B)$
$P(\text{Sparky} \mid \text{good job}) = P(\text{Sparky} \cap \text{good job}) \div P(\text{good job})$
$P(\text{Sparky} \mid \text{good job}) = (4/30 \div 19/30) = 4/19$

c. $P(\text{good job} \mid \text{Axle}) =$
$P(\text{good job} \cap \text{Axle}) \div P(\text{Axle}) =$
$1/2 \div 2/3 = 3/4$
(We didn't need to use the formula. $P(\text{good job} \mid \text{Axle})$ is given information. Axle does a good job 3 out of 4 times.)

More Car Trouble

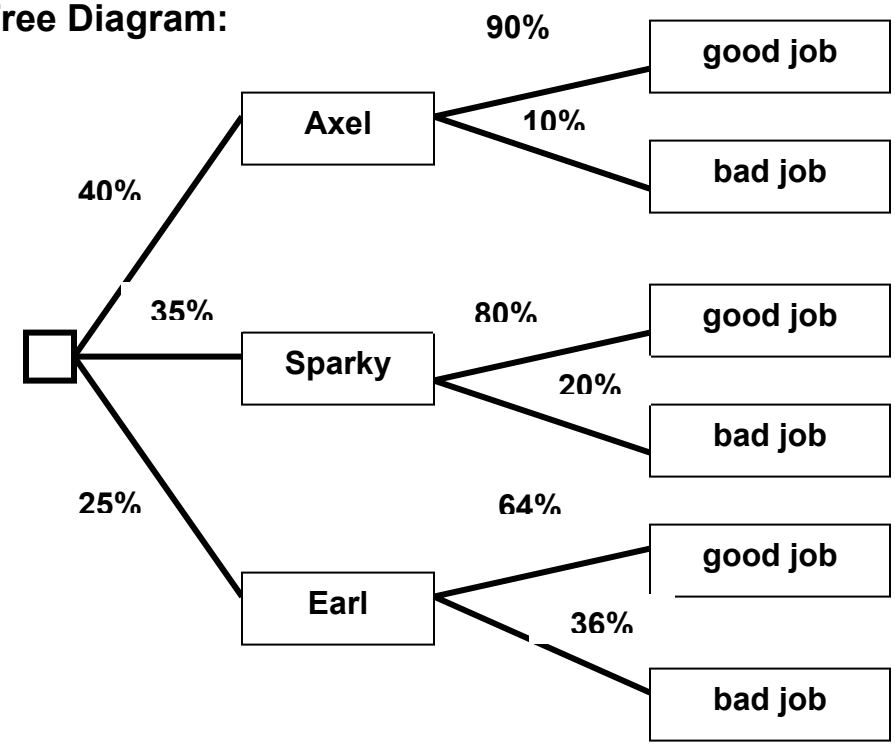
The local garage employs three mechanics, Axle, Sparky, and Earl but you never know which mechanic will be working on your car. Your neighborhood consumer club has found that Axle does 40% of the repairs, Sparky 35%, and Earl 25%. Axle does a good job nine out of ten times, Sparky does a good job eight out of ten times, and Earl does a good job only 64% of the time. If you plan to take your car in for repairs,

a. **what is the probability that a good job will be done?**

b. **what is the probability that Sparky worked on your car if a good job is done?**

c. **what is the probability that a good job is done if Axle worked on your car?**

Tree Diagram:



P (Axle ∩ good job) = 0.4 x 0.9 = 0.36

P (Axle ∩ bad job) = 0.4 x 0.1 = 0.04

P (Sparky ∩ good job) = 0.35 x 0.8 = 0.28

P (Sparky ∩ bad job) = 0.35 x 0.2 = 0.07

P (Earl ∩ good job) = 0.25 x 0.64 = 0.16

P (Earl ∩ bad job) = 0.25 x 0.36 = 0.09

Solution:

Use the formula for conditional probability which is
$P(A \mid B) = P(A \cap B) \div P(B)$

a. What is the probability that a good job will be done?

$P(\text{good job}) = P(\text{Axle} \cap \text{good job}) + P(\text{Sparky} \cap \text{good job}) + P(\text{Earl} \cap \text{good job}) = 0.36 + 0.28 + 0.16 = 0.80 \text{ or } 80\%$

b. What is the probability that Sparky worked on your car if a good job is done?

Use the formula for conditional probability which is
$P(A \mid B) = P(A \cap B) \div P(B)$
$P(\text{Sparky} \mid \text{good job}) = P(\text{Sparky} \cap \text{good job}) \div P(\text{good job})$
$P(\text{Sparky} \mid \text{good job}) = (0.28 \div 0.80) = 0.35$

Hence, "a good job being done" is independent of "Sparky working on the car".

$P(A \mid B) = P(A \cap B) \div P(B)$
$P(\text{Sparky} \mid \text{bad job}) = P(\text{Sparky} \cap \text{bad job}) \div P(\text{bad job})$
$P(\text{Sparky} \mid \text{bad job}) = (0.07 \div 0.20) = 0.35$

Likewise, "a bad job being done" is independent of "Sparky working on the car".

c. What is the probability that a good job is done if Axle worked on your car?

$P(\text{good job} \mid \text{Axle}) =$
$P(\text{good job} \cap \text{Axle}) \div P(\text{Axle}) =$
$0.36 \div 0.40 = 0.90$

(We didn't need to use the formula. $P(\text{good job} \mid \text{Axle})$ is given information. Axle does a good job 9 out of 10 times.)