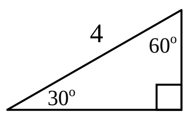
Trigonometric Ratios

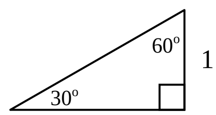
A. An Introduction to Trigonometric Ratios with 30^o-60^o-90^o Triangles

1. Fill in ALL the missing side lengths in the triangles below:

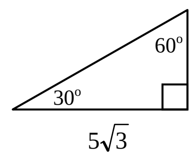
Triangle #1



Triangle #2



Triangle #3



2. Now fill in the numbers for the following ratios, using the 30^o angle in each triangle above as your reference angle. (It may help to imagine that you are standing at the vertex of the 30^o angle in each triangle as you fill in the chart.) Write your answers in simplest radical form (rationalize the denominators).

In relation to the 30^o Angle:	**Triangle #1**	**Triangle #2**	**Triangle #3**
$\frac{\text{Opposite Leg}}{\text{Hypotenuse}}$			
$\frac{\text{Adjacent Leg}}{\text{Hypotenuse}}$			
$\frac{\text{Opposite Leg}}{\text{Adjacent Leg}}$			

3. What do you notice about the ratios within any given row of the chart?

   ____

   When right triangles are similar, the above ratios remain constant for ANY given acute reference angle of ANY right triangle (not just 30^o-60^o-90^o triangles). If we were to know all three side lengths of three similar 37^o-53^o-90^o triangles (whose lengths are not so obvious and quick to compute!) and complete the above chart, say, for the 53^o angle in each triangle, we would still find that each row of the chart would give a constant ratio. Because these ratios are constant, they are a useful tool for finding missing angles and sides of right triangles (and countless other applications as well). These ratios are the basic Trigonometric ratios, and are defined below.

B. The Three Basic Trigonometric Ratios

The symbol θ, pronounced "theta", is a Greek letter which is commonly used in Trigonometry to represent an angle, and is used in the definitions below. Treat it as you would any other variable.

If θ is an acute angle of a right triangle, then:

Trigonometric Function		**Abbreviation**		**Ratio of the Following Lengths**
The sine of θ	=	$\sin(\theta)$	=	$\dfrac{\text{The leg opposite angle } \theta}{\text{The hypotenuse}}$
The cosine of θ	=	$\cos(\theta)$	=	$\dfrac{\text{The leg adjacent to angle } \theta}{\text{The hypotenuse}}$
The tangent of θ	=	$\tan(\theta)$	=	$\dfrac{\text{The leg opposite angle } \theta}{\text{The leg adjacent to angle } \theta}$

**Note: A useful mnemonic (in abbreviated form) for remembering the above chart is:*

SOH-CAH-TOA

SOH stands for **s**in(θ), **O**pposite, **H**ypotenuse: $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$

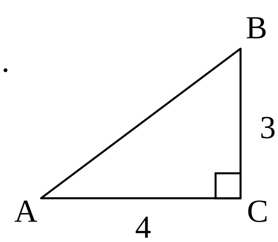
CAH stands for **c**os(θ), **A**djacent, **H**ypotenuse: $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA stands for **t**an(θ), **O**pposite, **A**djacent: $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

C. Examples

First find the missing side length of each triangle (by using the Pythagorean Theorem, or one of our theorems about Special Right Triangles, if applicable). Then find the indicated trigonometric ratios. *Pay close attention to which angle is being referenced!*

1.

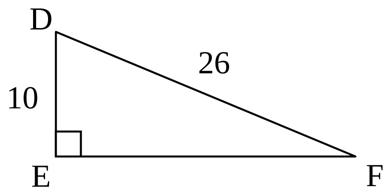


a) $\sin(A) =$ ______ d) $\sin(B) =$ ______

b) $\cos(A) =$ ______ e) $\cos(B) =$ ______

c) $\tan(A) =$ ______ f) $\tan(B) =$ ______

2.



a) $\sin(D) =$ ______

b) $\cos(D) =$ ______

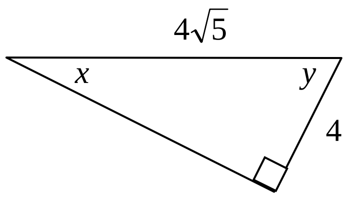
c) $\tan(D) =$ ______

d) $\sin(F) =$ ______

e) $\cos(F) =$ ______

f) $\tan(F) =$ ______

3.



a) $\sin(x) =$ ______

b) $\cos(x) =$ ______

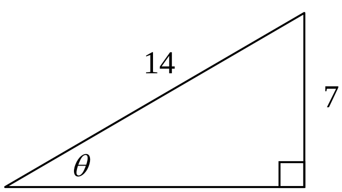
c) $\tan(x) =$ ______

d) $\sin(y) =$ ______

e) $\cos(y) =$ ______

f) $\tan(y) =$ ______

4.



a) $\sin(\theta) =$ ______

b) $\cos(\theta) =$ ______

c) $\tan(\theta) =$ ______