Rationalizing Denominators

A. Introduction

1. If we are performing a computation and obtain an answer of $\frac{3}{6}$, how are we likely to write that answer? _____

   Change the following fractions to simplest form:

   a) $\frac{12}{18} =$ ______ b) $\frac{60}{100} =$ ______

2. We are used to the convention of writing a fraction is in its simplest form. When fractions contain square roots, they also are generally expected to be written in a "standard" form. The standard in this case is that a fraction may not contain any radicals in the denominator. (The radicals that we will be working with are square roots.) When radicals do occur in the denominator (in fractions such as $\frac{2}{\sqrt{3}}$ or $\frac{1}{\sqrt{5}}$ or $\frac{\sqrt{6}}{\sqrt{7}}$), we must carry out a process known as rationalizing the denominator.

3. Equivalent Fractions
   In order to understand the mathematics behind rationalizing the denominator, we should look to a familiar example of writing equivalent fractions.

   If we were asked to write another fraction which is equivalent to $\frac{1}{2}$, we might, for example, write $\frac{5}{10}$. These fractions are equivalent since they have the same effect when used in calculations.

   We can see below how we mathematically obtain $\frac{5}{10}$ as an equivalent fraction to $\frac{1}{2}$:

   $$\frac{1}{2} \cdot \frac{5}{5} = \frac{5}{10}$$

   Notice that we multiplied by $\frac{5}{5}$,and $\frac{5}{5}$ equals 1. We can multiply any number by 1 without changing the value of the number.

4. Multiplying Square Roots
   Before we learn how to rationalize a denominator, we must also remember how to multiply square roots.

   Example: Multiply the following and simplify your answers.

   a) $\sqrt{7} \cdot \sqrt{7} =$ __________ b) $\sqrt{5} \cdot \sqrt{5} =$ __________
   c) $3\sqrt{2} \cdot \sqrt{2} =$ __________ d) $4\sqrt{3} \cdot \sqrt{3} =$ __________

B. Rationalizing the Denominator

1. Consider the fraction $\frac{1}{\sqrt{3}}$. To rationalize the denominator, we multiply by $\frac{\sqrt{3}}{\sqrt{3}}$, as follows:

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Thus, our final answer is $\frac{\sqrt{3}}{3}$.

Some may debate as to whether or not $\frac{\sqrt{3}}{3}$ looks any "simpler" than $\frac{1}{\sqrt{3}}$; nonetheless, this has become the standard. (Remember that your final answers ARE allowed to have square roots in the numerator, but your answers may NOT contain square roots in the denominator.)

2. Consider the fraction $\frac{4\sqrt{5}}{3\sqrt{7}}$. To rationalize the denominator, we multiply by $\frac{\sqrt{7}}{\sqrt{7}}$, as follows:

$$\frac{4\sqrt{5}}{3\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{35}}{3(7)} = \frac{4\sqrt{35}}{21}$$

Thus, our final answer is $\frac{4\sqrt{35}}{21}$.

3. Examples: Rationalize the denominators in the following fractions:

a) $\frac{1}{\sqrt{5}}$

b) $\frac{3}{\sqrt{10}}$

c) $\frac{6}{5\sqrt{2}}$

d) $\frac{2\sqrt{15}}{7\sqrt{2}}$