Polyhedra Patterns

Purpose:
Participants will build polyhedra using hands-on materials and use their properties to study patterns relating the vertices, edges, and faces.

Overview:
Participants will use coffee stirrers and play doh to build polyhedra. They will record their observations about the vertices, edges, and faces in a table and study the relationships among them. Their findings will be stated as a generalization known as Euler's rule or formula for solids. Further explorations will focus on the relationship between the vertices, edges, and face(s) of the two-dimensional faces of the polyhedra.

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.
V.016.D Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
V.016.E Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
V.017.D Understands how learning may be assisted through the use of mathematics manipulatives and technological tools.

TEKS Mathematics Objectives: The student is expected to:

4.8.C Describe shapes and solids in terms of vertices, edges, and faces.
4.14.D Use tools such as real objects, manipulatives, and technology to solve problems.
4.15.A Explain and record observations using objects, words, pictures, numbers, and technology.
4.16.A Make generalizations from patterns or sets of examples and nonexamples.
5.7.A Identify critical attributes including parallel, perpendicular, and congruent parts of geometric shapes and solids.
5.7.B Use critical attributes to define geometric shapes or solids.
5.14.D Use tools such as real objects, manipulatives, and technology to solve problems.
5.15.A Explain and record observations using objects, words, pictures, numbers, and technology.
5.16.A Make generalizations from patterns or sets of examples and nonexamples.
6.11.D Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
6.13.A Make conjectures from patterns or sets of examples and nonexamples.
7.6.C Use properties to classify solids, including pyramids, cones, prisms, and cylinders.
7.13.D Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
7.14.A Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.
7.15.A Make conjectures from patterns or sets of examples and nonexamples.
8.14.D Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
8.15.A Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.
8.16.A Make conjectures form patterns or sets of examples and nonexamples.

Terms.
Polyhedron, vertex, edge, face, tetrahedron, hexahedron, regular polyhedron, octahedron, dodecahedron, conjecture, generalization, genus, prism, pentagonal prism, pentagonal pyramid, Platonic solid

Materials.

- Coffee stirrers
- Play doh
- Scissors

Transparencies.

- *Polyhedra Patterns*

Activity Sheet(s).

- *Polyhedra Patterns*

References:

Gilbert, John E., Goodman, Tricia, et. al (1996). TEXTEAMS Geometry Institute: *Geometry from the High School Perspective* (pp. 1.50-1.58). Austin, TX: The University of Texas at Austin Charles A. Dana Center.

http://mathforum.org/Systems of Crystals

Links2Learning @ softhome.net

www.class.uidaho.edu/ngier/polyhedra/goc.htm

Procedure:

Steps	Questions/Math Notes
1. Have participants read the introduction to their investigation on Transparency *Polyhedra Patterns* . Review the definition of a polyhedron and ask for examples. Ask them to share other connections between polyhedra and the real-world based upon their experiences and knowledge.	Ask participants questions about their understanding of polyhedra or polyhedrons and their properties. *How would you describe the characteristics of a polyhedron?* *What are some common polyhedrons?* *How would you describe the faces of a tetrahedron? Hexahedron? Octahedron?* *How are the Platonic Solids different from the other polyhedrons? How are they alike?*
2. Share with participants that a Swiss mathematician named Leonhard Euler discovered a unique relationship among the vertices, faces, and edges of a convex polyhedron. Inform them that they will be investigating polyhedrons and generalizing their findings about Euler's discovery. Participants should work in groups of 2 to 4 on Activity *Polyhedra Patterns* using coffee stirrers and play doh at the vertices of their polyhedrons. It may be necessary to cut the stirrers as needed to build their structures. Monitor their work and ask questions to check and clarify their understanding of the investigations.	*How many polygons intersect at each vertex of a tetrahedron? Octahedron? Dodecahedron?* *What seems to be the least number of edges that meet at each vertex?* *How many faces share one edge in each of your polyhedrons?* *Which of the polyhedrons that you have built represent Platonic Solids? How do you know?* *What patterns do you observe in your table?*
3. Debrief the activities by having several groups present their findings on an overhead transparency.	*What is the relationship between the number of vertices, faces, and edges of a polyhedron?* *What is the relationship between the number of vertices, faces, and edges of any face of a polyhedron?* *How do these generalizations compare?* *Do you think these generalizations would hold true for any polyhedron? Explain using a counterexample.* *Do you think these generalizations would hold true for polyhedron with a hole? You will be*

	given an opportunity to explore and answer this question in the extension problem. *If a polyhedron has one hole, can the hole be cylindrical in shape? Why or why not?* *How do you think a hole in a polygonal face would affect the generalization made from table 2? Draw some examples and explore.*

Solution: Refer to the completed tables below.

Table 1

Polyhedron	Number of Faces	Number of Vertices	Number of Edges
Tetrahedron	4	4	6
Hexahedron	6	8	12
Octahedron	8	6	12
Pentagonal Prism	7	10	15
Pentagonal Pyramid	7	10	15
Triangular Prism	5	6	9

The relationship among the faces (F), vertices (V), and edges (E) for a convex polyhedron can be expressed as **F + V – E = 2.** This formula is often referred to as Euler's formula for solids with no holes. The number 2 can be described as an Euler characteristic.

Table 2

Shape of Polyhedron Face(s)	Number of Face(s)	Number of Vertices	Number of Edges
Triangle	1	3	3
Square/Rectangle	1	4	4
Pentagon	1	5	5
Hexagon	1	6	6

The relationship among the faces(F), vertices(V), and edges(E) for a polygon can be expressed as **F + V – E = 1.** The number 1 represents an Euler characteristic for polygons with no holes.

Extension:

1. Suppose a polyhedron has one hole. Investigate with a hexahedron, pentagonal prism, and

hexagonal prism. Write a rule that relates the number of vertices, faces, and edges for these solids with one hole. Such polyhedra are described as solids of genus one. If a solid has two holes, then it is said to have genus two.

2. How does the generalization for solids with no holes compare with your findings in problem 1?