Number of Divisors

Purpose:
Participants will investigate the relationship between the prime-factored form of a number and its total number of factors.

Overview:
In small groups, participants will generate the prime factorization of a number, list all the factors of the number, count the number of factors, and look for patterns. The goal is to find a connection between the prime-factored form of a number and its total number of factors.

TExES Mathematics 4-8 Competencies. The beginning teacher:

I.001.A Demonstrates an understanding of ideas from number theory (e.g., prime factorization, greatest common divisor) as they apply to whole numbers, integers, and rational numbers, and uses these ideas in problem situations.

TEKS Mathematics Objectives. The student is expected to:

4.4.C Recall and apply multiplication facts through 12 x 12.
4.4.D, 5.4.B Use multiplication to solve problems involving two-digit numbers.
4.4.E, 5.4.C Use division to solve problems involving one-digit divisors.
5.4.D Identify prime factors of a whole number and common factors of a set of whole numbers.
6.1.D Write prime factorizations using exponents.
6.1.E Identify factors and multiples including common factors and common multiples.

Terms.
Factors, prime factorization, exponents,

Materials.
- Transparencies
- Activity Sheets
- Calculators

Transparencies.
- *Number of Divisors*
- *Solutions*

Activity Sheets.
- *Number of Divisors*

Procedure:

Steps	Questions/Math Notes
1. Have participants work in small groups on *Number of Divisors Activity Sheet, Set A.*	*What connections do you see between the number in prime-factored form and the total number of factors?* (The number of factors is always one greater than the exponent of the prime factor. All of these numbers have only one prime factor.) *Explain why the number of factors is one greater than the exponent of the prime factor.* (The factors are p^0, p^1, p^2, p^3, …, p^n creating "n+1" number of factors.)
2. Have participants work in small groups on the remainder of the *Number of Divisors Activity Sheet.* Remind participants that their goal is to find a connection between the prime-factored form of the number and its total number of factors.	*What do the numbers in this set have in common?* *How are their prime-factored forms similar?* *How does their prime-factored form relate to the number of factors?*
3. Ask each group to make a chart of their findings for one of the Sets. Assign a different Set to each group.	*What patterns did you notice?* *Were there any variations to the patterns?*
4. Have each group discuss how they developed additional examples to fit the patterns they found.	*How did you generate additional examples that met the requirements of your pattern?*
5. Have one group summarize the rule that describes the connection between the prime-factored form of a number and its total number of factors.	*What connection is there between the prime-factored form of a number and its total number of factors?* (Once the number is in prime-factored form, take the exponents, increase each by one, and find the product of these numbers.)
6. Extend the concept.	*Find a number that has 20 factors.* (Possible answers will be in the form $p^4 \times q^3$ or $p^9 \times q^1$ or p^{19}. So a possible answer is $2^4 \times 3^3$ or 432.)

Solutions:

SET A			
#	Prime Factorization	List of All Factors	Number Of
4	2^2	1, 2, 4	3
8	2^3	1, 2, 4, 8	4
9	3^2	1, 3, 9	3
16	2^4	1, 2, 4, 8, 16	5
25	5^2	1, 5, 25	3
27	3^3	1, 3, 9, 27	4
32	2^5	1, 2, 4, 8, 16, 32	6
49	7^2	1, 7, 49	3
64	2^6	1, 2, 4, 8, 16, 32, 64	7
81	3^4	1, 3, 9, 27, 81	5
125	5^3	1, 5, 25, 125	4
128	2^7	1, 2, 4, 8, 16, 32, 64, 128	8

The total number of factors is always one larger than the exponent when the number is in prime-factored form.

SET B			
#	Prime Factorization	List of All Factors	Number Of Factors
6	$2^1 \times 3^1$	1, 2, 3, 6	4
15	$3^1 \times 5^1$	1, 3, 5, 15	4
77	$7^1 \times 11^1$	1, 7, 11, 77	4
Sample answers might be:			
10	$2^1 \times 5^1$	1, 2, 5, 10	4
14	$2^1 \times 7^1$	1, 2, 7, 14	4
21	$3^1 \times 7^1$	1, 3, 7, 21	4
35	$5^1 \times 7^1$	1, 5, 7, 35	4

1st pattern: To find the number of factors, take each exponent and increase it by one. Find the product of the two new numbers (2 x 2 = 4). The numbers in Set B have 4 factors.
2nd pattern: To find the number of factors, add the exponents together and double the sum. This pattern works for Sets B, C, and D but does not hold true for the other sets.

SET C			
#	Prime Factorization	List of All Factors	Number Of Factors
12	$2^2 \times 3^1$	1, 2, 3, 4, 6, 12	6
45	$3^2 \times 5^1$	1, 3, 5, 9, 15, 45	6
50	$2^1 \times 5^2$	1, 2, 5, 10, 25, 50	6
Sample answers might be:			
18	$2^1 \times 3^2$	1, 2, 3, 6, 9, 18	6
20	$2^2 \times 5^1$	1, 2, 4, 5, 10, 20	6
28	$2^2 \times 7^1$	1, 2, 4, 7, 14, 28	6

To find the number of factors, take each exponent and increase it by one. Find the product of the two new numbers (3 x 2 = 6). The numbers in Set C have 6 factors.

SET D			
#	Prime Factorization	List of All Factors	# of Factors
24	$2^3 \times 3^1$	1, 2, 3, 4, 6, 8, 12, 24	8
40	$2^3 \times 5^1$	1, 2, 4, 5, 8, 10, 20, 40	8
54	$2^1 \times 3^3$	1, 2, 3, 6, 9, 18, 27, 54	8
Sample answers might be:			
56	$2^3 \times 7^1$	1, 2, 4, 7, 8, 14, 28, 56	8
88	$2^3 \times 11^1$	1, 2, 4, 8, 11, 22, 44, 88	8
135	$3^3 \times 5^1$	1, 3, 5, 9, 15, 27, 45, 135	8

Take each exponent and increase it by one. Find the product of the two numbers (4 x 2 = 8).

SET E			
#	Prime Factorization	List of All Factors	# of Factors
72	$2^3 \times 3^2$	1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72	12
108	$2^2 \times 3^3$	1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108	12
200	$2^3 \times 5^2$	1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200	12
Sample answers might be:			
392	$2^3 \times 7^2$	1, 2, 4, 7, 8, 14, 28, 49, 56, 98, 196, 392	12
500	$2^2 \times 5^3$	1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500	12

Take each exponent and increase it by one. Find the product of the two numbers (4 x 3 = 12).

SET F			
#	Prime Factorization	List of All Factors	# Of Factors
36	$2^2 \times 3^2$	1, 2, 3, 4, 6, 9, 12, 18, 36	9
100	$2^2 \times 5^2$	1, 2, 4, 5, 10, 20, 25, 50, 100	9
225	$3^2 \times 5^2$	1, 3, 5, 9, 15, 25, 45, 75, 225	9
Sample answers might be:			
196	$2^2 \times 7^2$	1, 2, 4, 7, 14, 28, 49, 98, 196	9
441	$3^2 \times 7^2$	1, 3, 7, 9, 21, 49, 63, 147, 441	9

Take each exponent and increase it by one. Find the product of the two numbers (3 x 3 = 9).

SET G			
#	Prime Factorization	List of All Factors	# of Factors
144	$2^4 \times 3^2$	1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144	15
324	$2^2 \times 3^4$	1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 324	15
Sample answers might be:			
400	$2^4 \times 5^2$	1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400	15
784	$2^4 \times 7^2$	1, 2, 4, 7, 8, 14, 16, 28, 49, 56, 98, 112, 196, 392, 784	15

Take each exponent and increase it by one. Find the product of the two numbers (5 x 3 = 15).

SET H			
#	Prime Factorization	List of All Factors	# of Factors
30	$2^1 \times 3^1 \times 5^1$	1, 2, 3, 5, 6, 10, 15, 30	8
120	$2^3 \times 3^1 \times 5^1$	1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120	16
140	$2^2 \times 5^1 \times 7^1$	1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, 140	12
162	$2^1 \times 3^4$	1, 2, 3, 6, 9, 18, 27, 54, 81, 162	10
396	$2^2 \times 3^2 \times 11^1$	1, 2, 3, 4, 6, 9, 11, 12, 18, 22, 33, 36, 44, 66, 99, 132, 198, 396	18

Take each exponent and increase it by one. Find the product of the numbers.
The number of factors of 30 is equal to 2 x 2 x 2 or 8.
The number of factors of 120 is equal to 4 x 2 x 2 or 16.
The number of factors of 140 is equal to 3 x 2 x 2 or 12.
The number of factors of 162 is equal to 2 x 5 or 10.
The number of factors of 396 is equal to 3 x 3 x 2 or 18.