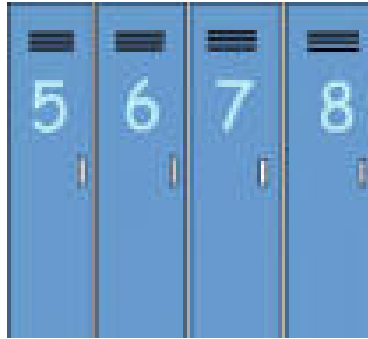


The Locker Problem



In Washington Middle School there are 1000 students and 1000 lockers, all shut and unlocked. Each year for homecoming the students line up in alphabetical order and perform the following strange homecoming ritual:

- The 1st student opens every locker.
- The 2nd student goes to every other locker (beginning with the 2nd locker) and closes it.
- The 3rd student *changes the state* of every third locker beginning with locker number 3. (If the locker is open the student closes it, and if the locker is closed the student opens it.)
- The 4th student *changes the state* of every 4th locker beginning with locker number 4.
- Continue this pattern until all 1000 students have performed their assigned ritual (ending with the 1000th student *changing the state* of the 1000th locker).

Your task is to determine:

1. How many lockers will be open? How many lockers will be closed?
2. Which lockers will be open and why?
3. Which locker(s) will be switched the most times? Why?
4. How many lockers, and which ones, will be switched exactly twice?