GCD and LCM

Purpose:
Participants will explore relationships between two numbers and their greatest common divisor and least common multiple.

Overview:
Pairs of participants will identify relationships between the product of two numbers and the product of the GCD and LCM of the two numbers. This investigation will require them to prime factor each of the two numbers, the GCD, and the LCM. Participants will look for patterns within the data.

TExES Mathematics 4-8 Competencies. The beginning teacher:

1.002.C	Uses a variety of concrete and visual representations to demonstrate the connections between operations and algorithms.
I.003.A	Demonstrates an understanding of ideas from number theory (e.g., prime factorization, greatest common divisor) as they apply to whole numbers, integers, and rational numbers, and uses these ideas in problem situations.
I.003.E	Applies properties of the real numbers to solve a variety of theoretical and applied problems.
1.004.A	Uses inductive reasoning to identify, extend, and create patterns using concrete models, figures, numbers, and algebraic expressions.
1.004.C	Makes, tests, validates, and uses conjectures about patterns and relationships in data presented in tables, sequences, or graphs.

TEKS Mathematics Objectives. The student is expected to:

4.4.B	Represent multiplication and division situations in picture, word, and number form.
4.4.C	Recall and apply mutiplication facts through 12 x 12.
4.4.D	Use multiplication to solve problems involving two-digit numbers.
4.4E	Use division to solve problems involving one-digit divisors.
5.3.B	Use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology).
5.3.C	Use division to solve problems involving whole numbers.
5.3.D	Identify prime factors of a whole number and common factors of a set of whole numbers.
5.5.B	Use lists, tables, charts, and diagrams to find patterns and make generalizations.
5.5.C	Identify prime and composite numbers using concrete models and patterns in factor pairs.
6.1.D	Use prime factorizations using exponents.
6.1.E	Identify factors and multiples including common factors and common multiples.
6.2.C	Use multiplication and division of whole numbers to solve problems.
6.5	Formulate an equation from a problem situation.
7.2.E	Simplify numerical expressions involving order of operations and exponents.
7.2.F	Select and use appropriate operations to solve problems and justify the selections.
8.2.A	Select and use appropriate operations to solve problems and justify the selections.

Terms.

Factor, divisor, multiple, exponent, prime factorization, prime factors, least common multiple, greatest common divisor

Materials.
- Transparencies
- Activity Sheets
- Calculator for each participant

Transparencies.
- *Transparency: GCD and LCM*
- *Transparency: Solution*

Activity Sheet(s).
- *Activity Sheet: GCD and LCM*

Procedure:

Steps	Questions/Math Notes
1. Display the transparency *GCD and LCM* on the overhead projector and allow participants time to ask questions about the table.	At first glance this activity appears to be a *plug and chug* exercise. Inform participants that the first few examples are fairly straightforward to allow participants to discover patterns. The latter problems are quite challenging and require a higher degree of understanding.
2. Introduce Venn Diagrams as a means for determining the GCD and LCM of two numbers.	See explanation of how to use Venn Diagrams after the Procedure section.
3. Have participants work in small groups on the Activity Sheet: *GCD and LCM* using calculators to assist them in identifying patterns. Encourage participants to find multiple answers for the last three problems.	*What information do you need to fill in each blank in the table?* *Is there additional information that you need?*
4. Have each group create a list of patterns they discovered. Have them post their lists on the wall.	*What patterns did you observe?* *How did you discover the pattern?*
5. Debrief the activity by having each group present a different problem.	*How can you determine the two numbers when you know their GCD and LCM?* *How can you determine the second number when all you know is one number and the LCM?*

**Using Venn Diagrams to Determine GCD and LCM of Two Numbers.**

Example: Find the GCD and LCM of 24 and 60.
Prime factor each number.
48 = 2 x 2 x 2 x 2 x 3 and 60 = 2 x 2 x 3 x 5
Draw two intersecting circles for your Venn Diagram.
Place the factors of 48 in one circle and the factors of 60 in the other circle with the common factors in the intersecting part of the circles. The GCD is product of the factors inside the intersecting part; the LCM is the product of all the factors in the Venn Diagram.
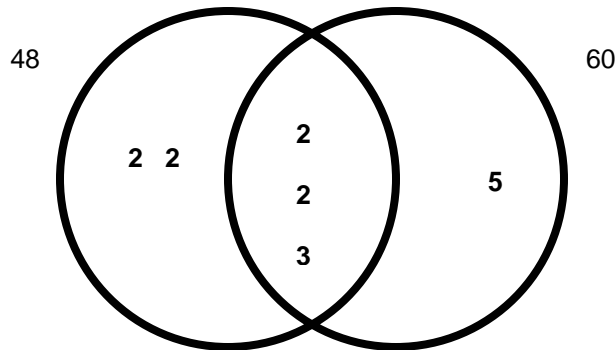
GCD (48, 60) = 2 x 2 x 3 = 12   LCM (48, 60) = 2 x 2 x 2 x 2 x 3 x 5 = 240

Supporting Website.
Visit Mathland website for an excellent explanation and visual representation of three useful methods for determining GCD and LCM of two numbers: (a) factor method, (b) Venn Diagram method, (c) set method.
http://math.youngzones.org/Math_2213_webpages/GCD_LCM.html

Solution.

A	B	A x B	GCD (A,B)	LCM (A,B)	GCD x LCM
1. 16	12	192	4	48	192
2. 25	40	1000	5	200	1000
3. 24	39	936	3	312	936
4. 88	66	5808	22	264	5808
5. 35	60	2100	5	420	2100
6. 50	45	2250	5	450	2250
7. 86	14	1204	2	602	1204
8. 6	72	432	6	72	432
18	24	432	6	72	432
9. 14	280	3920	14	280	3920
56	70	3920	14	280	3920
10. 72	112	8064	8	1008	8064
72	336	24192	24	1008	24192
72	1008	72576	72	1008	72576

List patterns you discovered.

A x B = GCD (A, B) x LCM (A, B)

For example, patterns emerge when solving problem #10:

A	B	A x B	GCD (A,B)	LCM (A,B)	GCD x LCM
72 2^3 x 3^2	112 2^4 x 3^0 x 7^1	8064 2^7 x 3^2 x 7^1	8 2^3	1008 2^4 x 3^2 x 7^1	8064 2^7 x 3^2 x 7^1
72 2^3 x 3^2	336 2^4 x 3^1 x 7^1	24192 2^7 x 3^3 x 7^1	24 2^3 x 3^1	1008 2^4 x 3^2 x 7^1	24192 2^7 x 3^3 x 7^1
72 2^3 x 3^2	1008 2^4 x 3^2 x 7^1	72576 2^7 x 3^4 x 7^1	72 2^3 x 3^2	1008 2^4 x 3^2 x 7^1	72576 2^7 x 3^4 x 7^1

Supporting Website.
Use the JavaScript GCD and LCM Calculator to devise problems for your students.
http://javascript.internet.com/calculators/lcm-and-gcd.html