# Chapter 2 <br> Polynomial and Rational Functions 

## Section 2.1: Linear and Quadratic Functions

> Linear Functions
$>$ Quadratic Functions

## Linear Functions

## Definition of a Linear Function:

A linear function is defined by an equation of the form

$$
f(x)=m x+b
$$

where $m$ and $b$ are constants.

## Graph of a Linear Function:

The graph of a linear function $f(x)=m x+b$ is a line with slope $m$ and $y$-intercept $b$.
(a) If $m=0$, the graph of $f$ is a horizontal line.
(b) If $m>0$, the graph of $f$ rises as we move left to right in the plane.
(c) If $m<0$, the graph of $f$ falls as we move left to right in the plane.

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If $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ are two points that lie on the graph of a linear function $f(x)=m x+b$, then the slope of the line is given by the slope formula

$$
m=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} .
$$



## Example:

Find the linear function $f$ satisfying the conditions that $f(-2)=1$ and $f(3)=2$.

## Solution:

The graph of $f$ is a line that passes through the points $(-2,1)$ and $(3,2)$. Use the slope formula to find the slope of the line.

$$
m=\frac{2-1}{3-(-2)}=\frac{1}{5}
$$

Recall that the point-slope form of a linear equation is given by

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

where the line passes through the point $\left(x_{1}, y_{1}\right)$ and the slope is $m$.
Now use the point-slope formula with $m=\frac{1}{5}$ and the point $(3,2)$ (or the point $(-2,1)$ ) to write an equation of the line.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) \\
y-2 & =\frac{1}{5}(x-3) \\
y-2 & =\frac{1}{5} x-\frac{3}{5} \\
y & =\frac{1}{5} x-\frac{3}{5}+2 \\
y & =\frac{1}{5} x+\frac{7}{5}
\end{aligned}
$$

Thus, $f(x)=\frac{1}{5} x+\frac{7}{5}$.

## Example:

Sketch the graph of the linear function $f(x)=\frac{1}{3} x+1$.

## Solution:

Identify the slope and $y$-intercept.
$m=\frac{1}{3} ; \quad b=1$

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To sketch the graph, begin by using the $y$-intercept to plot the point $(0,1)$.


Use the slope $\frac{1}{3}$ by moving 1 unit up and 3 units right from the point $(0,1)$ to locate another point on the line.


Pass a line through the two points.


The graph of the given function is shown below.


## Parallel and Perpendicular Lines:

Two lines with slopes $m_{1}$ and $m_{2}$ are parallel if and only if $m_{1}=m_{2}$.


Two lines with slopes $m_{1}$ and $m_{2}$ are perpendicular if and only if $m_{1} m_{2}=-1$.


## Example:

Find the linear function $f$ satisfying the conditions that $f(1)=2$ and the graph of $f$ is perpendicular to the line $3 x-y=7$.

## Solution:

The slope of the graph of $f$ must be the negative reciprocal of the slope of the line $3 x-y=7$. We must first find the slope of the line $3 x-y=7$.

Recall that the slope-intercept form of a linear equation is given by

$$
y=m x+b
$$

where the $y$-intercept is $b$ and the slope is $m$.

Thus, we solve the equation $3 x-y=7$ for $y$ to find the slope.

$$
\begin{aligned}
3 x-y & =7 \\
3 x-y-3 x & =7-3 x \\
-y & =7-3 x \\
(-1)(-y) & =(-1)(7-3 x) \\
y & =3 x-7
\end{aligned}
$$

The slope of the line $3 x-y=7$ is 3 . The negative reciprocal of 3 is $-\frac{1}{3}$.
Use the point-slope form to write an equation of the line with $m=-\frac{1}{3}$ and passing through the point $(1,2)$ since $f(1)=2$.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) \\
y-2 & =-\frac{1}{3}(x-1) \\
y-2 & =-\frac{1}{3} x+\frac{1}{3} \\
y-2+2 & =-\frac{1}{3} x+\frac{1}{3}+2 \\
y & =-\frac{1}{3} x+\frac{7}{3}
\end{aligned}
$$

Therefore, $f(x)=-\frac{1}{3} x+\frac{7}{3}$.

## Additional Example 1:

Find the linear function $f$ whose graph is shown below.


## Solution:

Determine the $y$-intercept from the graph.

$$
b=-1
$$



Determine the slope from the graph.

$$
m=\frac{2}{1}=2
$$



Recall that a linear function is defined by an equation of the form $f(x)=m x+b$, where $m$ is the slope and $b$ is the $y$-intercept. To write the linear function, substitute 2 for $m$ and -1 for $b$ in the equation $f(x)=m x+b$.

Thus, $f(x)=2 x-1$.

## Additional Example 2:

Find the linear function $f$ whose graph passes through the points $(3,-1)$ and $(-2,4)$.

## Solution:

The graph of $f$ is a line that passes through the points $(3,-1)$ and $(-2,4)$. Find the slope of the line by substituting $x_{1}=3, y_{1}=-1, x_{2}=-2$, and $y_{2}=4$ into the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{4-(-1)}{-2-3} \\
& =\frac{4+1}{-5} \\
& =\frac{5}{-5} \\
& =-1
\end{aligned}
$$

Use the point-slope formula to find an equation of the line with slope -1 that passes through the point $(3,-1)$.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) \\
y-(-1) & =-1(x-3) \\
y+1 & =-x+3 \\
y & =-x+2
\end{aligned}
$$

Thus, $f(x)=-x+2$.

## Additional Example 3:

Find the linear function $f$ whose graph is the line with $x$-intercept -5 and $y$-intercept 4 .

## Solution:

The graph of the line with $x$-intercept -5 and $y$-intercept 4 is shown below.


Find the slope of the line by substituting $x_{1}=-5, y_{1}=0, x_{2}=0$, and $y_{2}=4$ into the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{4-0}{0-(-5)} \\
& =\frac{4}{0+5} \\
& =\frac{4}{5}
\end{aligned}
$$

Substitute $m=\frac{4}{5}$ and $b=4$ into the equation $f(x)=m x+b$.

Thus, $f(x)=\frac{4}{5} x+4$.

## Additional Example 4:

Find the linear function $f$ whose graph is the line that passes through the point
$(1,3)$ and is parallel to the line with equation $y=-\frac{1}{2} x+1$.

## Solution:

A line that is parallel to the line $y=-\frac{1}{2} x+1$ must have the same slope as this line. Thus, the slope of the required line is $-\frac{1}{2}$.

Substitute $m=-\frac{1}{2}, x_{1}=1$, and $y_{1}=3$ into the point-slope form.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) \\
y-3 & =-\frac{1}{2}(x-1) \\
y-3 & =-\frac{1}{2} x+\frac{1}{2} \\
y & =-\frac{1}{2} x+\frac{7}{2}
\end{aligned}
$$

Thus, $f(x)=-\frac{1}{2} x+\frac{7}{2}$.

## Quadratic Functions

## Definition of a Quadratic Function:

A quadratic function $f$ is a function that can be put in the form

$$
f(x)=a x^{2}+b x+c,
$$

where $a, b$, and $c$ are constants with $a \neq 0$.

## Graph of a Quadratic Function:

A quadratic function $f$ can be expressed in the form

$$
f(x)=a(x-h)^{2}+k,
$$

called the standard form, by the method of completing the square.

The graph of $f$ is a parabola with vertex $(h, k)$. The axis of symmetry is the vertical line $x=h$.

If $a>0$, the parabola opens upward and the vertex $(h, k)$ is the lowest point on the graph. The minimum value of $f$ is $f(h)=k$.


$$
f(x)=a(x-h)^{2}+k, a>0
$$

If $a<0$, the parabola opens downward and the vertex $(h, k)$ is the highest point on the graph. The maximum value of $f$ is $f(h)=k$.


$$
f(x)=a(x-h)^{2}+k, a<0
$$

## Example:

Write the quadratic function $f(x)=2 x^{2}+4 x+3$ in standard form by completing the square. Identify the vertex, the axis of symmetry, and the maximum or minimum value. Then sketch the graph.

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## Solution:

$$
\begin{aligned}
f(x) & =2 x^{2}+4 x+3 \\
& =2\left(x^{2}+2 x\right)+3 \quad \text { Factor out a } 2 \text { from the first two terms. } \\
& =2\left(x^{2}+2 x+1\right)+3-2 \text { Complete the square for } x^{2}+2 x \text { by adding }\left[\frac{1}{2} \cdot 2\right]^{2}=1 . \\
& =2(x+1)^{2}+1
\end{aligned}
$$

The function in standard form is $f(x)=2(x+1)^{2}+1$.

The vertex is the point $(-1,1)$.

The axis of symmetry is the vertical line $x=-1$.

Since the coefficient of $x^{2}$ is positive, the vertex is the lowest point on the graph.

The minimum value of $f$ is $f(-1)=1$. The function has no maximum value.

To graph the function $f(x)=2(x+1)^{2}+1$, begin with the graph of $g(x)=x^{2}$.


Shift the graph of $g$ to the left one unit to obtain the graph of $y=(x+1)^{2}$.


Stretch the graph of $y=(x+1)^{2}$ vertically by a factor of 2 to obtain the graph of $y=2(x+1)^{2}$.


Shift the graph of $y=2(x+1)^{2}$ upward 1 unit to obtain the graph of $f(x)=2(x+1)^{2}+1$.


The graph of $f$ is shown below.


## Using Formulas to Find the Vertex:

If a quadratic function is given in the form $f(x)=a x^{2}+b x+c$, then the vertex $(h, k)$ can be found by using the formulas $h=-\frac{b}{2 a}$ and $k=f\left(-\frac{b}{2 a}\right)$.

## Example:

Given the quadratic function $f(x)=-x^{2}-3 x+3$, find the vertex and the maximum or minimum value.

## Solution:

For $f(x)=-x^{2}-3 x+3, a=-1$, and $b=-3$.
$h=-\frac{b}{2 a}=-\frac{-3}{2(-1)}=-\frac{-3}{-2}=-\frac{3}{2}$
$k=f\left(-\frac{3}{2}\right)=-\left(-\frac{3}{2}\right)^{2}-3\left(-\frac{3}{2}\right)+3=-\frac{9}{4}+\frac{9}{2}+3=-\frac{9}{4}+\frac{18}{4}+\frac{12}{4}=\frac{21}{4}$
The vertex is the point $\left(-\frac{3}{2}, \frac{21}{4}\right)$.

Since the coefficient of $x^{2}$ is negative, the vertex is the highest point on the graph.

The maximum value of $f$ is $f\left(-\frac{3}{2}\right)=\frac{21}{4}$.

## Intercepts of the Graph of a Quadratic Function:

## $x$-intercepts:

Recall that an $x$-intercept of the graph of a function $y=f(x)$ is the first coordinate of a point where the graph crosses the $x$-axis. To find the $x$-intercepts, solve the equation $f(x)=0$.

For a quadratic function $f(x)=a x^{2}+b x+c$, the $x$-intercepts of the parabola are found by solving the quadratic equation $a x^{2}+b x+c=0$. Methods for solving a quadratic equation include factoring, completing the square, and using the quadratic formula. Recall the quadratic formula:

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} .
$$

The expression $b^{2}-4 a c$ is called the discrimin ant.
(a) If $b^{2}-4 a c>0$, the equation $a x^{2}+b x+c=0$ has two distinct real solutions which are the two $x$-intercepts of the graph of the quadratic function $f(x)=a x^{2}+b x+c$.
(b) If $b^{2}-4 a c=0$, the equation $a x^{2}+b x+c=0$ has exactly one real solution which is the one $x$-intercept of the graph of the quadratic function $f(x)=a x^{2}+b x+c$.
(c) If $b^{2}-4 a c<0$, the equation $a x^{2}+b x+c=0$ has no real solution and the graph of the quadratic function $f(x)=a x^{2}+b x+c$ has no $x$-intercept.




## $y$-intercept:

Recall that the $y$-intercept of the graph of a function $y=f(x)$ is the second coordinate of the point where the graph intersects the $y$-axis. To find the $y$-intercept, find $f(0)$.

For a quadratic function $f(x)=a x^{2}+b x+c$, the $y$-intercept is $f(0)=c$.

## Example:

For the quadratic function $f(x)=2 x^{2}+3 x-2$, find the the vertex, the axis of symmetry, the maximum or minimum value, the $y$-intercept and all $x$-intercepts.
Then sketch a graph that includes the above information.

## Solution:

For $f(x)=2 x^{2}+3 x-2, a=2, b=3$, and $c=-2$.
$h=-\frac{b}{2 a}=-\frac{3}{2(2)}=-\frac{3}{4}$
$k=f\left(-\frac{b}{2 a}\right)=f\left(-\frac{3}{4}\right)=2\left(-\frac{3}{4}\right)^{2}+3\left(-\frac{3}{4}\right)-2=2\left(\frac{9}{16}\right)-\frac{9}{4}-2=\frac{9}{8}-\frac{9}{4}-2=-\frac{25}{8}$
The vertex is the point $\left(-\frac{3}{4},-\frac{25}{8}\right)$.
The axis of symmetry is the vertical line $x=-\frac{3}{4}$.

Since the coefficient of $x^{2}$ is positive, the vertex is the lowest point on the graph.
The minimum value of $f$ is $f\left(-\frac{3}{4}\right)=-\frac{25}{8}$.

The $y$-intercept $f(0)=-2$.

To find the $x$-intercepts, solve the quadratic equation $2 x^{2}+3 x-2=0$.

This equation can be solved by factoring:

Note: For a review of factoring, please refer to Appendix A.1: Factoring Polynomials.

$$
\begin{aligned}
2 x^{2}+3 x-2=0 & \\
(2 x-1)(x+2)=0 & \text { Factor the left-hand side. } \\
2 x-1=0 & \text { or } x+2=0
\end{aligned} \text { U se the zero-product property. }
$$

The $x$-intercepts are $x=\frac{1}{2}$ and $x=-2$.

The graph is shown below.


## Additional Example 1:

Write the quadratic function $f(x)=x^{2}+2 x-3$ in standard form by completing the square. Identify the vertex, the axis of symmetry, and the maximum or minimum value. Then sketch the graph.

## Solution:

Complete the square for $x^{2}+2 x$ by adding $\left[\frac{1}{2} \cdot 2\right]^{2}=1$.

$$
\begin{aligned}
f(x) & =x^{2}+2 x-3 \\
& =\left(x^{2}+2 x+1\right)-3-1 \\
& =(x+1)^{2}-4
\end{aligned}
$$

The function in standard form is $f(x)=(x+1)^{2}-4$.

The vertex is the point $(-1,-4)$.

The axis of symmetry is the vertical line $x=-1$.

Since the coefficient of $x^{2}$ is positive, the vertex is the lowest point on the graph.
The minimum value of the function is $f(-1)=-4$. There is no maximum value.
To graph the function, begin with the graph of $g(x)=x^{2}$ shown below.


First shift the graph of $g 1$ unit to the left.
$\left[f(x)=(x+1)^{2}-4\right]$


Now shift 4 units downward.
$\left[f(x)=(x+1)^{2}-4\right]$


The graph of the given function is shown below.


## Additional Example 2:

Write the quadratic function $f(x)=-x^{2}+4 x-1$ in standard form by completing the square. Identify the vertex, the axis of symmetry, the maximum or minimum value. Then sketch the graph.

## Solution:

Factor out -1 from the first two terms.
$f(x)=-\left(x^{2}-4 x\right)-1$
Complete the square for $x^{2}-4 x$ by adding $\left[\frac{1}{2} \cdot(-4)\right]^{2}=4$.

$$
\begin{aligned}
f(x) & =-x^{2}+4 x-1 \\
& =-\left(x^{2}-4 x\right)-1 \\
& =-\left(x^{2}-4 x+4\right)-1+4 \\
& =-(x-2)^{2}+3
\end{aligned}
$$

The function in standard form is $f(x)=-(x-2)^{2}+3$.

The vertex is the point $(2,3)$.

The axis of symmetry is the vertical line $x=2$.

Since the coefficient of $x^{2}$ is negative, the vertex is the highest point on the graph.
The maximum value of the function is $f(2)=3$. There is no minimum value.
To graph the function, begin with the graph of $g(x)=x^{2}$ shown below.


First shift the graph of $g 2$ units to the right.
$\left[f(x)=-(x-2)^{2}+3\right]$


Next reflect in the $x$-axis.
$\left[f(x)=-(x-2)^{2}+3\right]$


Now shift 3 units upward.
$\left[f(x)=-(x-2)^{2}+3\right]$


The graph of the given function is shown below.


## Additional Example 3:

Find a quadratic function satisfying the given conditions. The vertex is the point $(-3,4)$ and $(1,8)$ is another point on the graph.

Solution:
Substitute $h=-3$ and $k=4$ in the standard form of a quadratic function

$$
\begin{aligned}
f(x) & =a(x-h)^{2}+k \\
& =a(x-(-3))^{2}+4 \\
& =a(x+3)^{2}+4
\end{aligned}
$$

$f(1)=8$ (We are given that the point $(1,8)$ is on the graph.)

We must determine $a$. First substitute $x=1$ in the standard form.

$$
\begin{aligned}
f(x) & =a(x+3)^{2}+4 \\
f(1) & =a(1+3)^{2}+4 \\
& =a(4)^{2}+4 \\
& =16 a+4
\end{aligned}
$$

Since $f(1)=8$, substitute 8 for $f(1)$.

$$
\begin{aligned}
f(1) & =16 a+4 \\
8 & =16 a+4
\end{aligned}
$$

Solve the equation $8=16 a+4$ for $a$.

$$
\begin{aligned}
8 & =16 a+4 \\
8-4 & =16 a+4-4 \\
4 & =16 a \\
\frac{4}{16} & =\frac{16 a}{16} \\
\frac{1}{4} & =a
\end{aligned}
$$

Now substitute $a=\frac{1}{4}$ in the standard form $f(x)=a(x+3)^{2}+4$.

The quadratic funtion is $f(x)=\frac{1}{4}(x+3)^{2}+4$.

## Additional Example 4:

For $f(x)=-2 x^{2}+9 x+3$, find the vertex $(h, k)$ by using the formulas $h=-\frac{b}{2 a}$ and $k=f\left(-\frac{b}{2 a}\right)$. Then identify the maximum or minimum value of the function.

## Solution:

For $f(x)=-2 x^{2}+9 x+3, a=-2$ and $b=9$.
To find $h$, substitute $a=-2$ and $b=9$ in the formula $h=-\frac{b}{2 a}$.

$$
\begin{aligned}
h & =-\frac{b}{2 a} \\
& =-\frac{9}{2(-2)} \\
& =-\frac{9}{-4} \\
& =\frac{9}{4}
\end{aligned}
$$

$k=f\left(-\frac{b}{2 a}\right)=f\left(\frac{9}{4}\right)$.

$$
f(x)=-2 x^{2}+9 x+3
$$

$$
k=f\left(\frac{9}{4}\right)=-2\left(\frac{9}{4}\right)^{2}+9\left(\frac{9}{4}\right)+3
$$

$$
=-2\left(\frac{81}{16}\right)+\frac{81}{4}+3
$$

$$
=-\frac{81}{8}+\frac{81}{4}+3
$$

$$
=-\frac{81}{8}+\frac{162}{8}+\frac{24}{8}
$$

$$
=\frac{105}{8}
$$

The vertex is the point $\left(\frac{9}{4}, \frac{105}{8}\right)$.

Since the coefficient of $x^{2}$ is negative, the vertex is the highest point on the graph.
The maximum value of the function is $f\left(\frac{9}{4}\right)=\frac{105}{8}$. There is no minimum value.

## Additional Example 5:

For the quadratic function $f(x)=x^{2}+x-6$, find the vertex, the axis of symmetry, the maximum or minimum value, the $y$-intercept, and all $x$-intercepts. Then sketch a graph that includes the above information.

## Solution:

$$
\begin{aligned}
& \text { For } f(x)=x^{2}+x-6, a=1, b=1 \\
& \begin{aligned}
& h=-\frac{b}{2 a}=-\frac{1}{2(1)}=-\frac{1}{2} \\
& \begin{aligned}
k=f\left(-\frac{1}{2}\right) & =\left(-\frac{1}{2}\right)^{2}+\left(-\frac{1}{2}\right)-6 \\
& =\frac{1}{4}-\frac{1}{2}-6 \\
& =\frac{1}{4}-\frac{2}{4}-\frac{24}{4} \\
& =-\frac{25}{4}
\end{aligned}
\end{aligned} .
\end{aligned}
$$

The vertex is the point $\left(-\frac{1}{2},-\frac{25}{4}\right)$.
The axis of symmetry is the vertical line $x=-\frac{1}{2}$.
Since the coefficient of $x^{2}$ is positive, the vertex is the lowest point on the graph.
The minimum value of the function is $f\left(-\frac{1}{2}\right)=-\frac{25}{4}$. There is no maximum value.

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The $y$-intercept is $f(0)=-6$.

To find the $x$-intercepts, solve the quadratic equation $x^{2}+x-6=0$.

$$
\begin{gathered}
x^{2}+x-6=0 \\
(x+3)(x-2)=0 \\
x+3=0 \quad \text { or } x-2=0 \\
x=-3 \text { or } x=2
\end{gathered}
$$

The $x$-intercepts are -3 and 2 .

The graph is shown below.


## Exercise Set 2.1: Linear and Quadratic Functions

Find the slope of the line that passes through the following points. If it is undefined, state 'Undefined.'

1. $(-2,3)$ and $(6,-7)$
2. $(-1,-6)$ and $(-5,10)$
3. $(8,-7)$ and $(-1,-7)$
4. $(3,-8)$ and $(3,-4)$

Find the slope of each of the following lines.
5. $c$
6. $d$
7. $e$
8. $f$


Find the linear function $f$ which corresponds to each graph shown below.
9.

10.


For each of the following equations,

## Exercise Set 2.1: Linear and Quadratic Functions

30. Passes through $(5,-7)$; perpendicular to the line $y=-5 x+3$
31. Passes through (2,3); parallel to the line $5 x-2 y=6$
32. Passes through $(-1,5)$; parallel to the line $4 x+3 y=8$
33. Passes through $(2,3)$; perpendicular to the line $5 x-2 y=6$
34. Passes through $(-1,5)$; perpendicular to the line $4 x+3 y=8$
35. Passes through (4, -6); parallel to the line containing ( $3,-5$ ) and $(2,1)$
36. Passes through $(8,3)$; parallel to the line containing $(-2,-3)$ and $(-4,6)$
37. Perpendicular to the line containing ( $4,-2$ ) and $(10,4)$; passes through the midpoint of the line segment connecting these points.
38. Perpendicular to the line containing $(-3,5)$ and $(7,-1)$; passes through the midpoint of the line segment connecting these points.
39. $f$ passes through $(-3,-6)$ and $f^{-1}$ passes through $(-8,-9)$.
40. $f$ passes through $(2,-1)$ and $f^{-1}$ passes through $(9,4)$.
41. The $x$-intercept for $f$ is 3 and the $x$-intercept for $f^{-1}$ is -8 .
42. The $y$-intercept for $f$ is 4 and the $y$-intercept for $f^{-1}$ is -6 .

Answer the following, assuming that each situation can be modeled by a linear function.
43. If a company can make 21 computers for $\$ 23,000$, and can make 40 computers for $\$ 38,200$, write an equation that represents the cost of $x$ computers.
44. A certain electrician charges a $\$ 40$ traveling fee, and then charges $\$ 55$ per hour of labor. Write an equation that represents the cost of a job that takes $x$ hours.

## For each of the quadratic functions given below:

(a) Complete the square to write the equation in the standard form $f(x)=a(x-h)^{2}+k$.
(b) State the coordinates of the vertex of the parabola.
(c) Sketch the graph of the parabola.
(d) State the maximum or minimum value of the function, and state whether it is a maximum or a minimum.
(e) Find the axis of symmetry. (Be sure to write your answer as an equation of a line.)
45. $f(x)=x^{2}+6 x+7$
46. $f(x)=x^{2}-8 x+21$
47. $f(x)=x^{2}-2 x$
48. $f(x)=x^{2}+10 x$
49. $f(x)=2 x^{2}-8 x+11$
50. $f(x)=3 x^{2}+18 x+15$
51. $f(x)=-x^{2}-8 x-9$
52. $f(x)=-x^{2}+4 x-7$
53. $f(x)=-4 x^{2}+24 x-27$
54. $f(x)=-2 x^{2}-8 x-14$
55. $f(x)=x^{2}-5 x+3$
56. $f(x)=x^{2}+7 x-1$
57. $f(x)=2-3 x-4 x^{2}$
58. $f(x)=7-x-3 x^{2}$

## Exercise Set 2.1: Linear and Quadratic Functions

Each of the quadratic functions below is written in the form $f(x)=a x^{2}+b x+c$. For each function:
(a) Find the vertex $(h, k)$ of the parabola by using the formulas $h=-\frac{b}{2 a}$ and $k=f\left(-\frac{b}{2 a}\right)$.
(Note: When only the vertex is needed, this method can be used instead of completing the square.)
(b) State the maximum or minimum value of the function, and state whether it is a maximum or a minimum.
59. $f(x)=x^{2}-12 x+50$
60. $f(x)=-x^{2}+14 x-10$
61. $f(x)=-2 x^{2}+16 x-9$
62. $f(x)=3 x^{2}-12 x+29$
63. $f(x)=-2 x^{2}+9 x+3$
64. $f(x)=-6 x^{2}+x-5$

The following method can be used to sketch a reasonably accurate graph of a parabola without plotting points. Each of the quadratic functions below is written in the form $f(x)=a x^{2}+b x+c$. The graph of a quadratic function is a parabola with vertex, where $h=-\frac{b}{2 a}$ and $k=f\left(-\frac{b}{2 a}\right)$.
(a) Find all $x$-intercept(s) of the parabola by setting $f(x)=0$ and solving for $x$.
(b) Find the $y$-intercept of the parabola.
(c) Give the coordinates of the vertex $(h, k)$ of the parabola, using the formulas $h=-\frac{b}{2 a}$ and $k=f\left(-\frac{b}{2 a}\right)$.
(d) State the maximum or minimum value of the function, and state whether it is a maximum or a minimum.
(e) Find the axis of symmetry. (Be sure to write your answer as an equation of a line.)
(f) Draw a graph of the parabola that includes the features from parts (a) through (d).
65. $f(x)=x^{2}-2 x-15$
66. $f(x)=x^{2}-8 x+16$
67. $f(x)=3 x^{2}+12 x-36$
68. $f(x)=-2 x^{2}+16 x+40$
69. $f(x)=-4 x^{2}-8 x+5$
70. $f(x)=4 x^{2}-16 x-9$
71. $f(x)=x^{2}-6 x+3$
72. $f(x)=x^{2}+10 x+5$
73. $f(x)=x^{2}-2 x+5$
74. $f(x)=x^{2}+4$
75. $f(x)=9-4 x^{2}$
76. $f(x)=9 x^{2}-100$

For each of the following problems, find a quadratic function satisfying the given conditions.
77. Vertex $(2,-5)$; passes through $(7,70)$
78. Vertex $(-1,-8)$; passes through $(2,10)$
79. Vertex $(5,7)$; passes through $(3,4)$
80. Vertex $(-4,3)$; passes through $(1,13)$

Answer the following.
81. Two numbers have a sum of 10 . Find the largest possible value of their product.
82. Jim is beginning to create a garden in his back yard. He has 60 feet of fence to enclose the rectangular garden, and he wants to maximize the area of the garden. Find the dimensions Jim should use for the length and width of the garden. Then state the area of the garden.
83. A rocket is fired directly upwards with a velocity of $80 \mathrm{ft} / \mathrm{sec}$. The equation for its height, $H$, as a function of time, $t$, is given by the function

$$
H(t)=-16 t^{2}+80 t
$$

(a) Find the time at which the rocket reaches its maximum height.
(b) Find the maximum height of the rocket.

## Exercise Set 2.1: Linear and Quadratic Functions

84. A manufacturer has determined that their daily profit in dollars from selling $x$ machines is given by the function

$$
P(x)=-200+50 x-0.1 x^{2} .
$$

Using this model, what is the maximum daily profit that the manufacturer can expect?

