Section 1.2: Functions and Graphs

- Graphing a Function
- Additional Properties of Functions and Their Graphs

Graphing a Function

Definition:

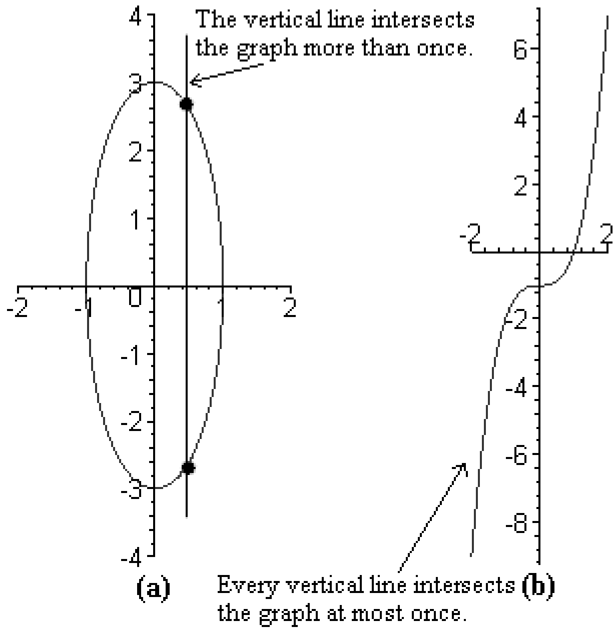
The graph of a function f is the set of all points (x, y) in the coordinate plane where the x-coordinates are elements of the domain of f and where the corresponding y-coordinates are given by $y = f(x)$.

For a function $y = f(x)$, exactly one y-value can be assigned to a given x-value. This means that a vertical line can intersect the graph of a function at most once. This provides a test for determining whether or not a curve in the xy-plane is the graph of a function.

The Vertical Line Test:

A curve in the xy-plane is the graph of a function if and only if no vertical line intersects the curve more than once.

For the figure shown below, using the Vertical Line Test, we see that the curve in part (a) does not represent a function while the curve in part (b) does represent a function.



Example:

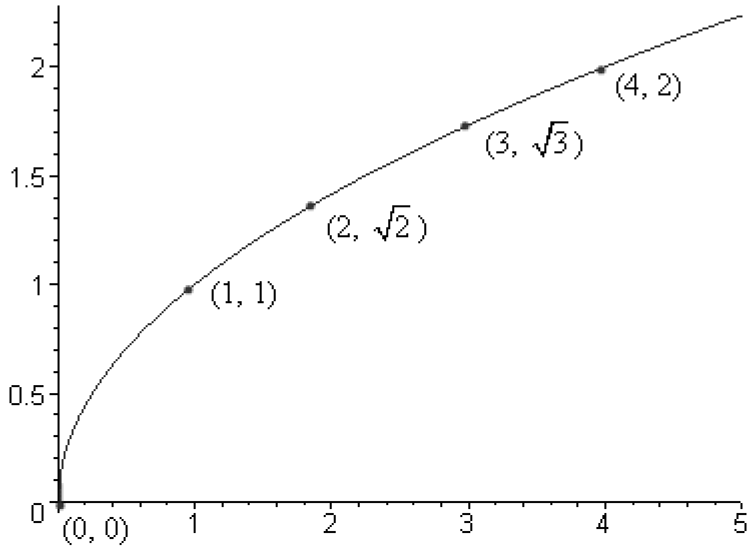
Sketch the graph of the function $f(x) = \sqrt{x}$.

Solution:

The function f is not defined for those values of x that are negative. Thus, the domain of the function is $\{x \mid x \geq 0\}$. The domain in interval notation is $[0, \infty)$. To graph the function, make a table of values.

x	$f(x)$	$(x, f(x))$
0	$f(0) = \sqrt{0} = 0$	$(0,0)$
1	$f(1) = \sqrt{1} = 1$	$(1,1)$
2	$f(2) = \sqrt{2}$	$(2, \sqrt{2})$
3	$f(3) = \sqrt{3}$	$(3, \sqrt{3})$
4	$f(4) = \sqrt{4} = 2$	$(4,2)$

The graph is shown in the figure below.



Example:

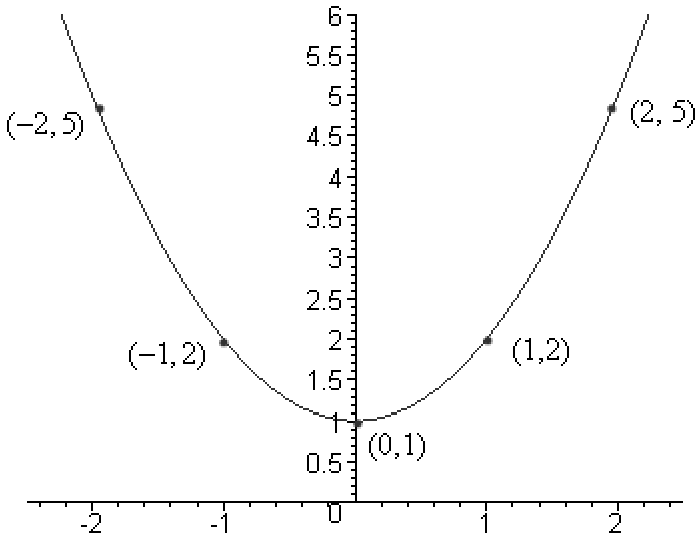
Sketch the graph of the function $f(x) = x^2 + 1$.

Solution:

The function f is defined for all values of x. The domain of f is the set of real numbers. The domain in interval notation is $(-\infty, \infty)$. The graph is a parabola. To graph the function, make a table of values.

x	$f(x)$	$(x, f(x))$
-2	$f(-2) = (-2)^2 + 1 = 4 + 1 = 5$	$(-2, 5)$
-1	$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$	$(-1, 2)$
0	$f(0) = 0^2 + 1 = 0 + 1 = 1$	$(0, 1)$
1	$f(1) = 1^2 + 1 = 1 + 1 = 2$	$(1, 2)$
2	$f(2) = 2^2 + 1 = 4 + 1 = 5$	$(2, 5)$

The graph is shown in the figure below.



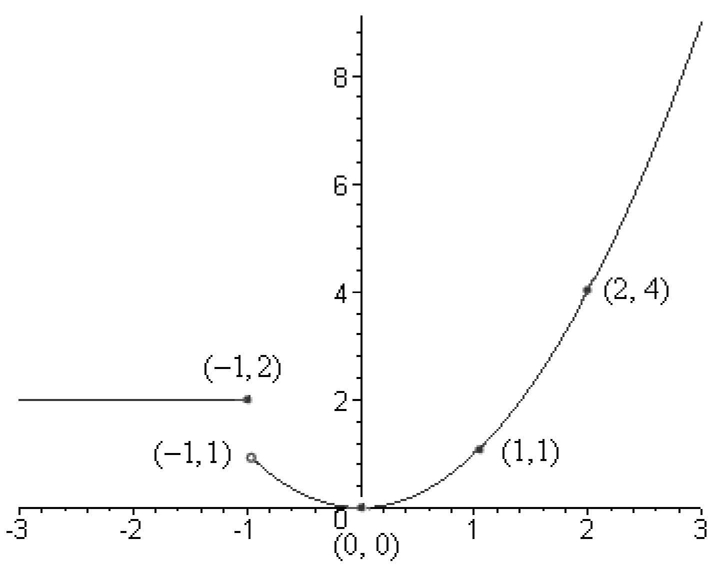
Example:

Sketch the graph of the function $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$.

Solution:

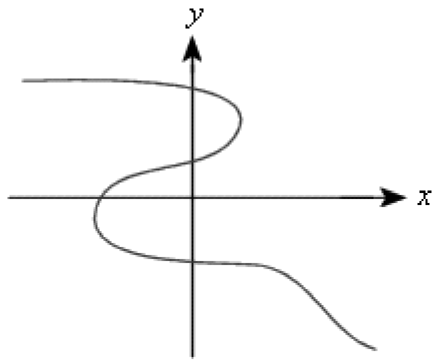
For $x \le -1$, the graph coincides with the horizontal line $y = 2$. For $x > -1$, the graph coincides with the graph of the parabola $y = x^2$.

The graph is shown in the figure below. The open dot at $(-1, 1)$ indicates that this point is excluded from the graph.

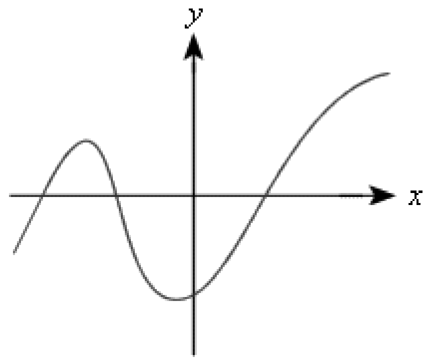


Additional Example 1:

State whether or not each of the following graphs represents a function.



(a)

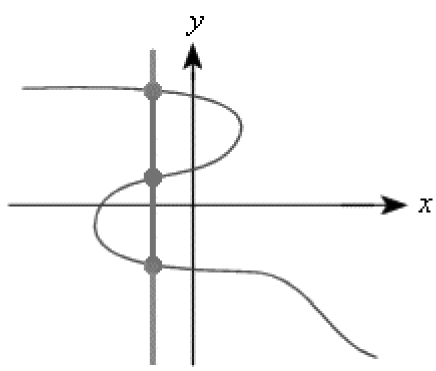


(b)

Solution:

Part (a):

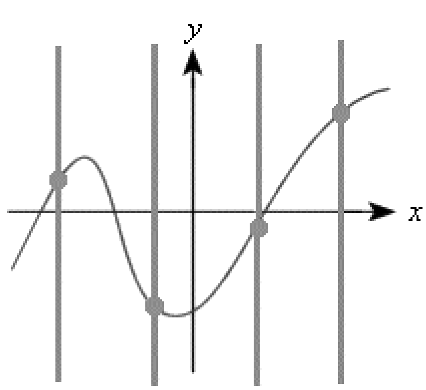
Use the Vertical Line Test for the graph in part (a).



The first graph does not represent a function since we can find a vertical line that intersects the graph more than once.

Part (b):

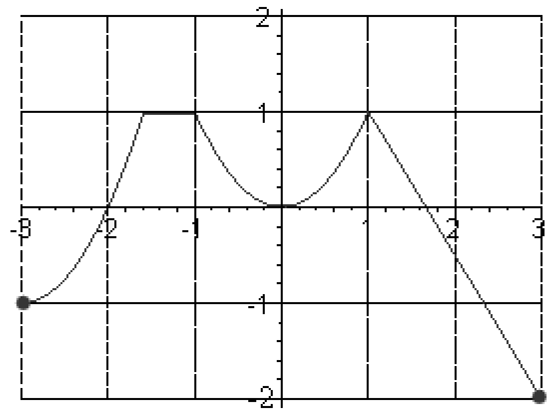
Use the Vertical Line Test for the graph in part (b).



It is easy to see (from the 4 example lines shown) that every vertical line intersects the graph at most once. The second graph represents a function.

Additional Example 2:

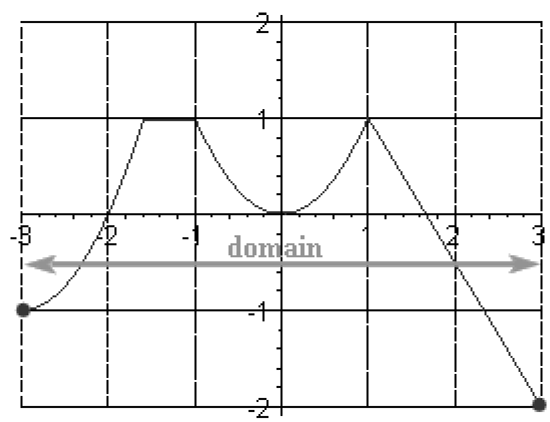
The graph of $y = f(x)$ is shown below. (a) Find the domain of f. (b) Find the range of f. (c) Find the following function values: $f(-3)$; $f(-1)$; $f(0)$; $f(1)$. (d) For what value(s) of x is $f(x) = -2$?



Solution:

Part (a):

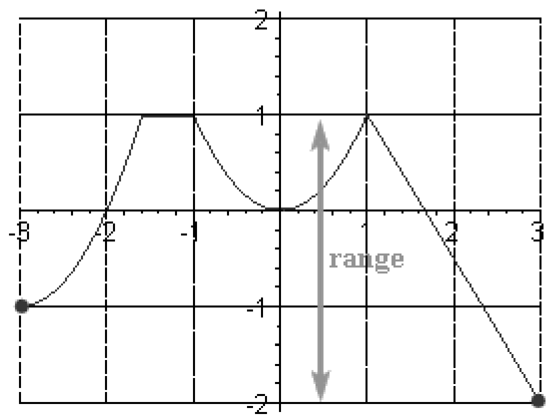
Determine the domain by inspecting the graph.



The domain in interval notation is $[-3,3]$.

Part (b):

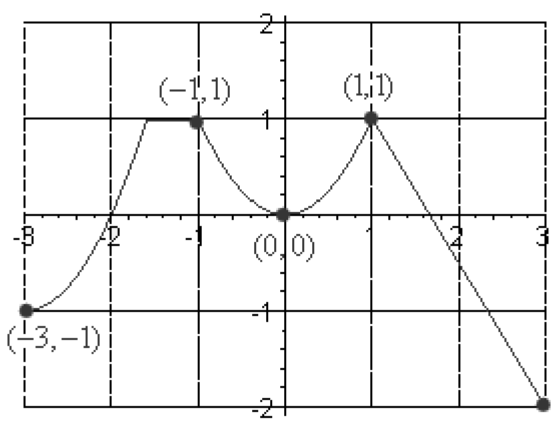
Determine the range by inspecting the graph.



The range in interval notation is $[-2, 1]$.

Part (c):

Label the points on the graph that have first coordinates of $-3, -1, 0,$ and 1.

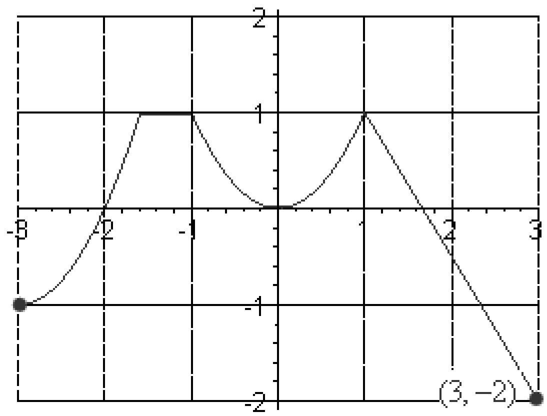


Find the function values $f(-3)$, $f(-1)$, $f(0)$, and $f(1)$ by selecting the second coordinates of the points labeled on the graph:

$f(-3) = -1$
$f(-1) = 1$
$f(0) = 0$
$f(1) = 1$

Part (d):

Label the points on the graph that have a second coordinate of -2.



The value of x for which $f(x) = -2$ is found by selecting the first coordinate of the point labeled on the graph:

$x = 3$

Additional Example 3:

State the domain of the function $f(x) = |x| + 2$ in interval notation and then sketch the graph of the function.

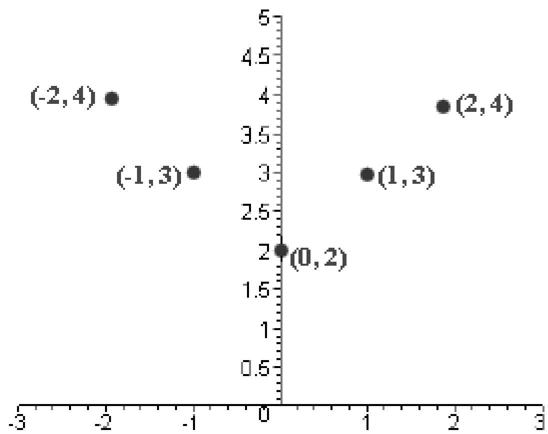
Solution:

For each real number x, $|x| + 2$ is a real number. Thus, the domain in interval notation is $(-\infty, \infty)$.

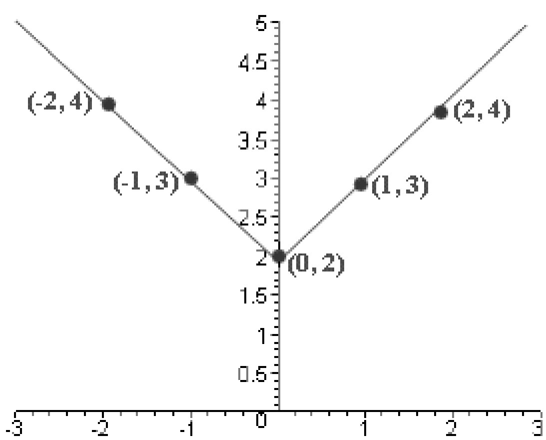
To sketch the graph, make a table of values.

x	$f(x)$	$(x, f(x))$
-2	$f(-2) = \|-2\| + 2 = 2 + 2 = 4$	$(-2, 4)$
-1	$f(-1) = \|-1\| + 2 = 1 + 2 = 3$	$(-1, 3)$
0	$f(0) = \|0\| + 2 = 0 + 2 = 2$	$(0, 2)$
1	$f(1) = \|1\| + 2 = 1 + 2 = 3$	$(1, 3)$
2	$f(2) = \|2\| + 2 = 2 + 2 = 4$	$(2, 4)$

Plot the points found in the table.



Plot additional points, if necessary, to sketch the graph.



Additional Example 4:

State the domain of the function $f(x) = -x^2 + 2$ in interval notation and then sketch the graph of the function.

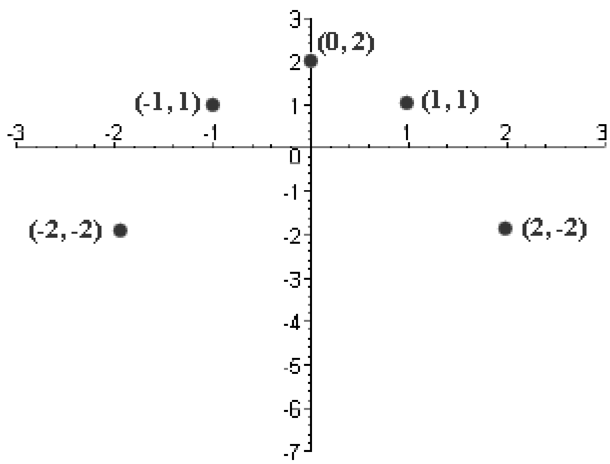
Solution:

For each real number x, $-x^2 + 2$ is a real number. Thus, the domain in interval notation is $(-\infty, \infty)$.

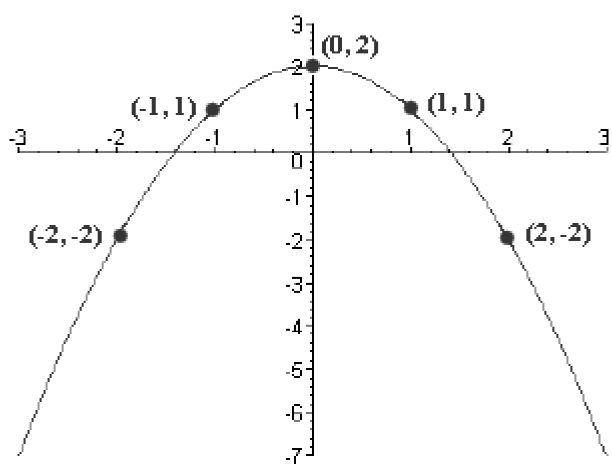
To sketch the graph, make a table of values.

x	$f(x)$	$(x, f(x))$
-2	$f(-2)=-(-2)^2+2=-4+2=-2$	$(-2,-2)$
-1	$f(-1)=-(-1)^2+2=-1+2=1$	$(-1,1)$
0	$f(0)=-(0)^2+2=2$	$(0,2)$
1	$f(1)=-(1)^2+2=-1+2=1$	$(1,1)$
2	$f(2)=-(2)^2+2=-4+2=-2$	$(2,-2)$

Plot the points found in the table.



Plot additional points, if necessary, to sketch the graph.



Additional Example 5:

State the domain of the function $f(x)=\sqrt{x+2}$ in interval notation and then sketch the graph of the function.

Solution:

For $\sqrt{x+2}$ to be a real number, $x+2$ cannot be negative. Solve the inequality $x+2 \geq 0$ to find the domain of f.

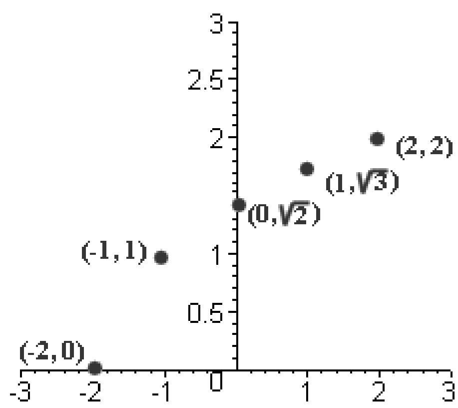
$$x+2 \geq 0$$
$$x+2-2 \geq 0-2$$
$$x \geq -2$$

The domain in interval notation is $[-2, \infty)$.

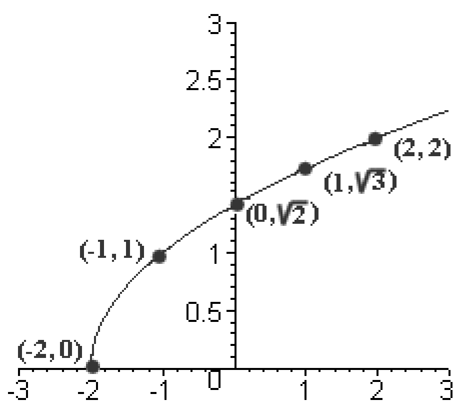
To sketch the graph, make a table of values.

x	$f(x)$	$(x, f(x))$
-2	$f(-2)=\sqrt{-2+2}=\sqrt{0}=0$	$(-2,0)$
-1	$f(-1)=\sqrt{-1+2}=\sqrt{1}=1$	$(-1,1)$
0	$f(0)=\sqrt{0+2}=\sqrt{2}$	$(0,\sqrt{2})$
1	$f(1)=\sqrt{1+2}=\sqrt{3}$	$(1,\sqrt{3})$
2	$f(2)=\sqrt{2+2}=\sqrt{4}=2$	$(2,2)$

Plot the points found in the table.



Plot additional points, if necessary, to sketch the graph.



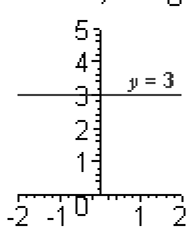
Additional Example 6:

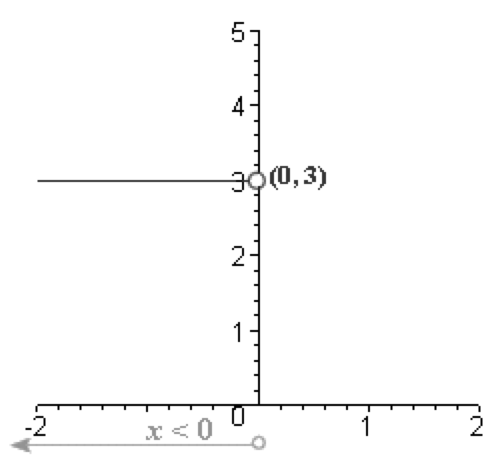
Sketch the graph of the piecewise-defined function f given by

$$f(x) = \begin{cases} 3 & \text{if } x < 0 \\ x^2 + 1 & \text{if } x \geq 0 \end{cases}.$$

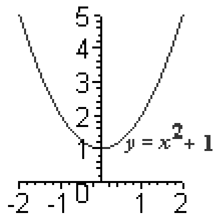
Solution:

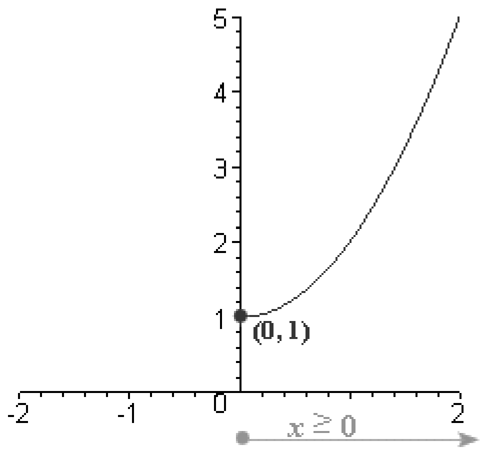
For $x < 0$, the graph of f coincides with the horizontal line $y = 3$.



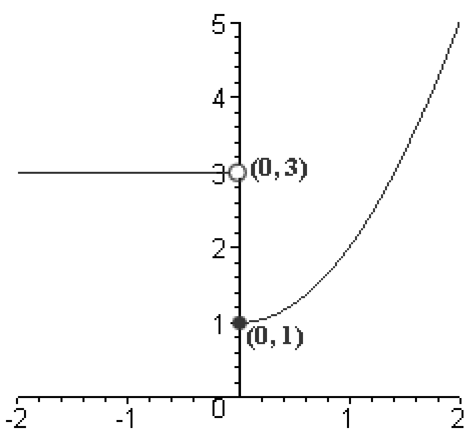


For $x \geq 0$, the graph of f coincides with the parabola $y = x^2 + 1$.





Sketch the graph.

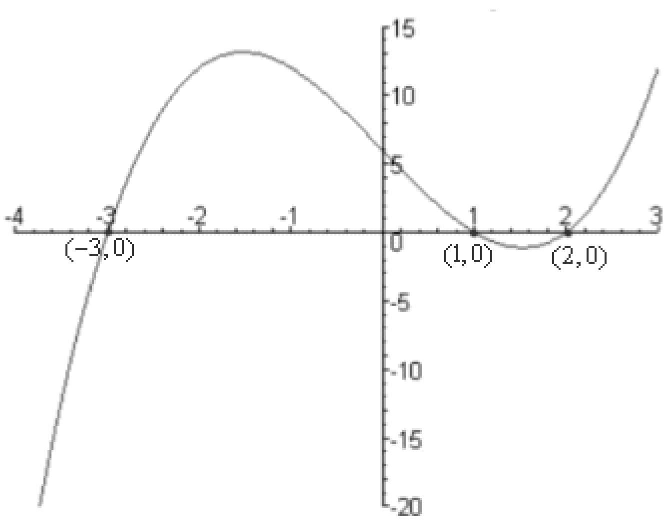


Additional Properties of Functions and Their Graphs

An *x*-intercept of the Graph of a Function:

An <u>x-intercept</u> of the graph of a function $y = f(x)$ is the <u>first</u> coordinate of a point where the graph intersects the x-axis.

The graph of a function is shown below.

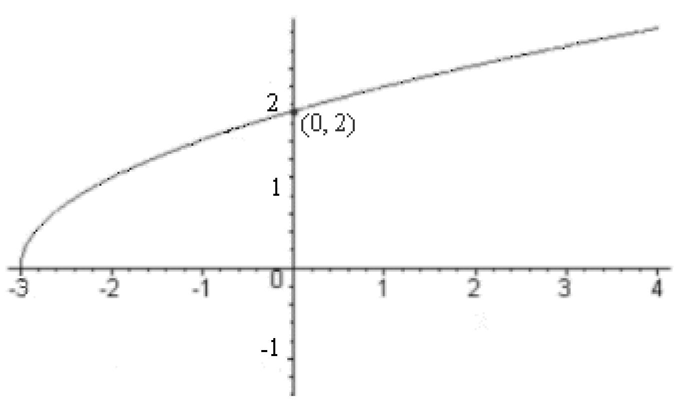


There are three x-intercepts: $-3, 1,$ and 2.

The *y*-intercept of the Graph of a Function:

The <u>y-intercept</u> of the graph of a function $y = f(x)$ is the <u>second</u> coordinate of a point where the graph intersects the y-axis.

The graph of a function is shown below.



The y-intercept is 2.

Finding the Intercepts of the Graph of a Function:

To find the x-intercepts of the graph of a function $y = f(x)$, set $y = 0$ into the equation and solve for x; that is, solve the equation $f(x) = 0$.

To find the y-intercept of the graph, find $f(0)$.

Example:
Find the intercepts of the graph of the function $f(x) = 4 - x^2$.

Solution:
To find the x-intercepts, solve the equation $f(x) = 0$ for x.

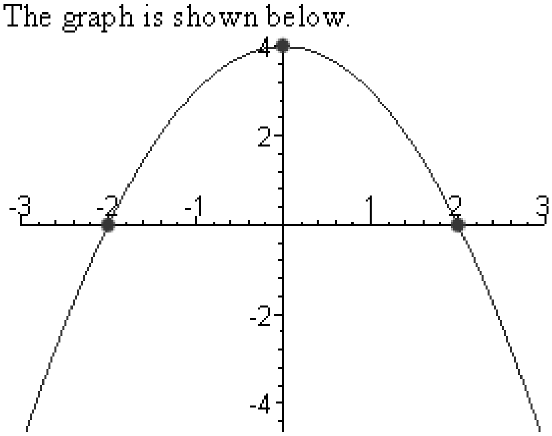
$$\begin{aligned} f(x) &= 0 \\ 4 - x^2 &= 0 \\ 4 &= x^2 \\ \pm\sqrt{4} &= x \\ \pm 2 &= x \end{aligned}$$

The x-intercepts are 2 and -2.
To find the y-intercept, find $f(0)$.

$$\begin{aligned} f(x) &= 4 - x^2 \\ f(0) &= 4 - 0^2 \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

The y-intercept is 4.

The graph is shown below.

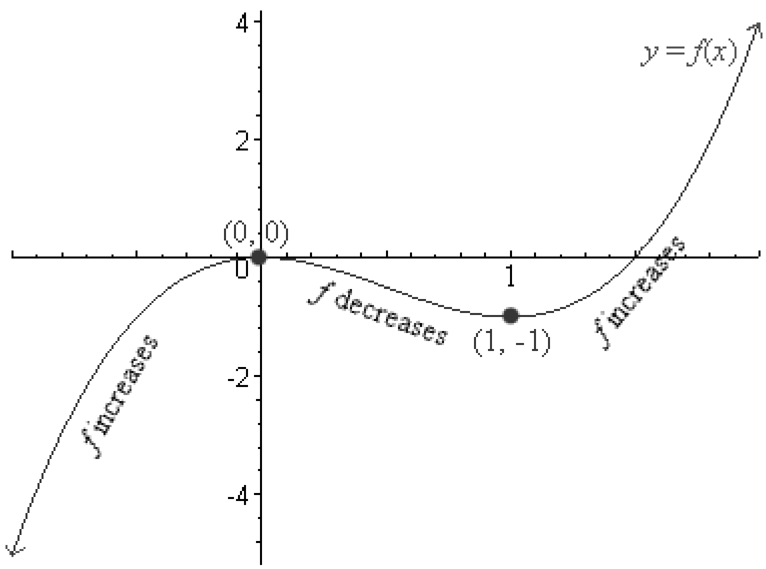


Increasing/Decreasing Functions:

Let I be an interval of the following form: $(-\infty,\infty)$ or $(-\infty,b)$ or (a,∞) or (a,b).

A function f is said to be increasing on I if functional values $f(x)$ increase as x increases on the interval I. In this case, the graph of f rises as x increases on the interval I.

A function f is said to be decreasing on I if functional values $f(x)$ decrease as x increases on the interval I. In this case, the graph of f falls as x increases on the interval I.



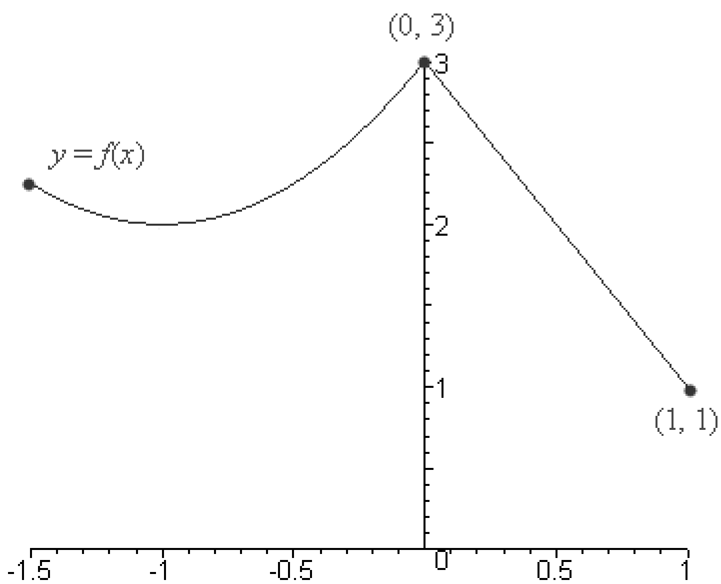
The function is increasing on the intervals $(-\infty,0)$ and $(1,\infty)$. Note that the graph of f is rising on these intervals.

The function is decreasing on the interval $(0,1)$. Note that the graph of f is falling on this interval.

Maximum/Minimum Values of a Function:

A function f is said to have a maximum at a number c in its domain if $f(x) \leq f(c)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f. If a function f has a maximum value, then the graph of f has a highest point. The second coordinate of a highest point is the maximum value.

A function f is said to have a minimum at a number c in its domain if $f(x) \geq f(c)$ for all x in the domain of f. The number $f(c)$ is called the minimum value of f. If a function f has a minimum value, then the graph of f has a lowest point. The second coordinate of a lowest point is the minimum value.



The function f whose graph is shown above has domain $[-3/2, 1]$ and range $[1, 3]$. The function has both a maximum value and a minimum value.

Note that $f(x) \leq f(0) = 3$ for all x in the interval $[-3/2, 1]$. The highest point on the graph is the point $(0, 3)$. The maximum value of f is $3 = f(0)$.

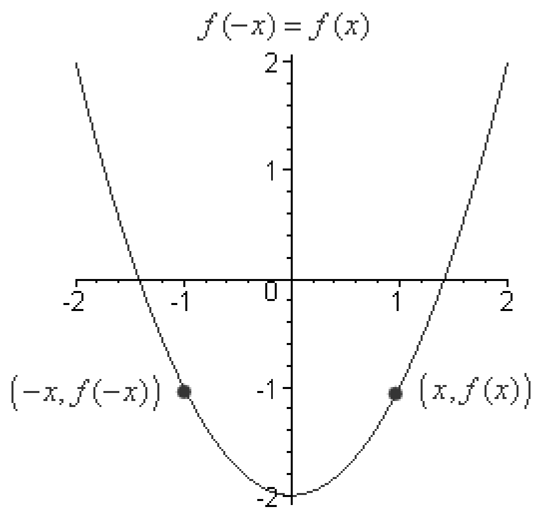
Note also that $f(x) \geq f(1) = 1$ for all x in the interval $[-3/2, 1]$. The lowest point on the graph is the point $(1, 1)$. The minimum value of f is $1 = f(1)$.

Even/Odd Functions:

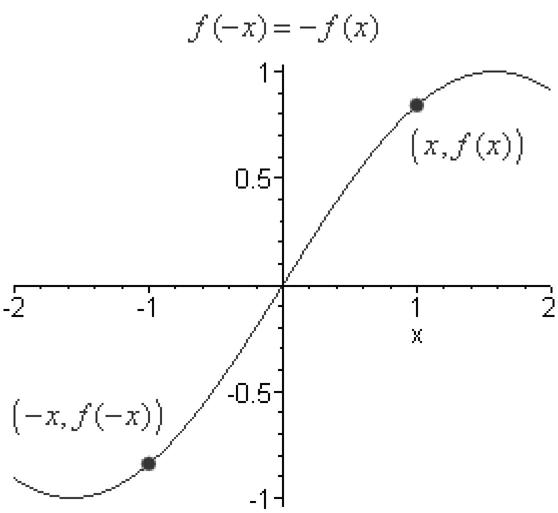
A function f is even if $f(-x) = f(x)$ for all x in the domain of f. The graph of an even function is symmetric with respect to the y-axis. This means that the point $(-a, f(a))$ is on the graph of the function whenever the point $(a, f(a))$ is on the graph.

A function f is odd if $f(-x) = -f(x)$ for all x in the domain of f. The graph of an odd function is symmetric with respect to the origin. This means that the point $(-a, -f(a))$ is on the graph of the function whenever the point $(a, f(a))$ is on the graph.

The graph shown below is the graph of an even function.



The graph shown below is the graph of an odd function.



Example:

Determine whether the function $f(x) = \frac{1}{x^2}$ is even, odd, or neither. If the function is even or odd, use symmetry to sketch its graph.

Solution:

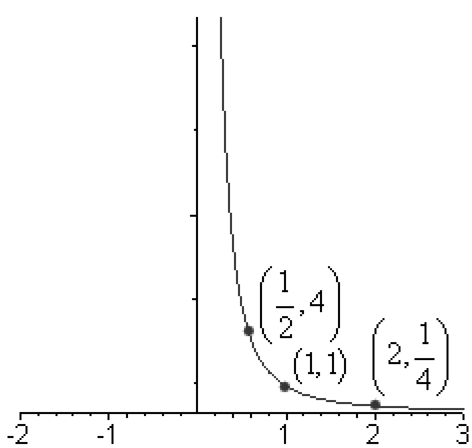
$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

Since $f(-x) = f(x)$, the function f is even.

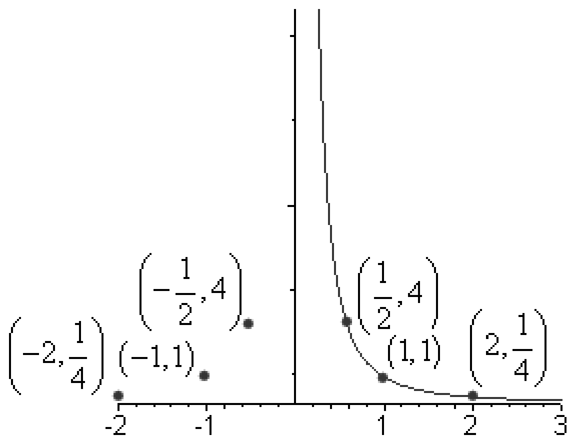
The domain of the function is $(-\infty, 0) \cup (0, \infty)$. Begin by making a table of values by choosing values of x for which $x > 0$.

x	$f(x)$	$(x, f(x))$
$\frac{1}{2}$	$f\left(\frac{1}{2}\right) = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$	$\left(\frac{1}{2}, 4\right)$
1	$f(1) = \frac{1}{1^2} = 1$	$(1, 1)$
2	$f(2) = \frac{1}{2^2} = \frac{1}{4}$	$\left(2, \frac{1}{4}\right)$

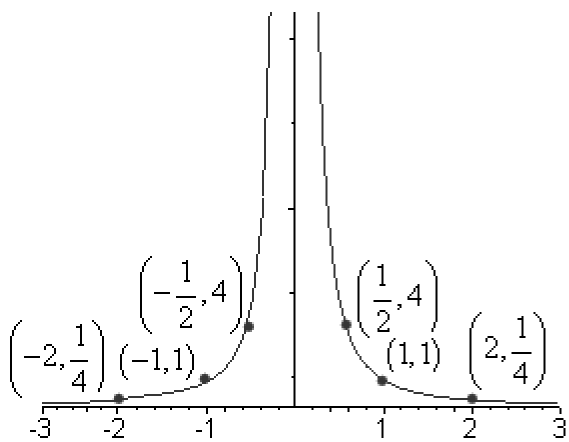
Plot the points shown in the table and plot additional points, if necessary, to show the portion of the graph for $x > 0$.



Use symmetry to find other points on the graph. Since the function is even, its graph is symmetric with respect to the y-axis.



Sketch the graph.



Additional Example 1:

Find the intercepts of the graph of the function $f(x)=\frac{4}{5}x+\frac{8}{5}$.

Solution:

To find the x-intercepts, solve the equation $f(x)=0$ for x.

$$f(x)=0$$

$$\frac{4}{5}x+\frac{8}{5}=0$$

$$\frac{4}{5}x+\frac{8}{5}-\frac{8}{5}=0-\frac{8}{5}$$

$$\frac{4}{5}x = -\frac{8}{5}$$

$$\frac{\cancel{5}}{\cancel{4}} \cdot \frac{\cancel{4}}{\cancel{5}} x = \frac{\cancel{5}}{4}\left(-\frac{8}{\cancel{5}}\right)$$

$$x = -\frac{8}{4}$$

$$x = -2$$

To find the y-intercept, find $f(0)$.

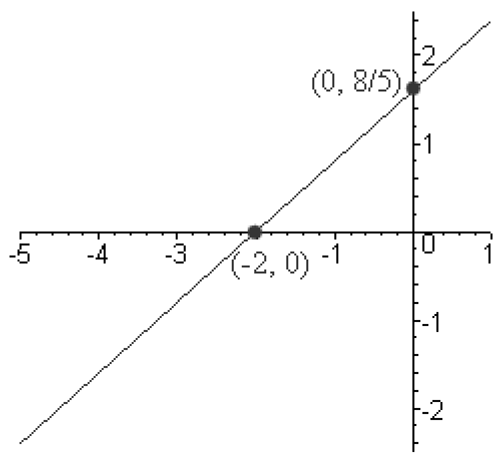
$$f(x) = \frac{4}{5}x + \frac{8}{5}$$

$$f(0) = \frac{4}{5} \cdot 0 + \frac{8}{5}$$

$$= 0 + \frac{8}{5}$$

$$= \frac{8}{5}$$

The x-intercept is -2 and the y-intercept is $\frac{8}{5}$. The graph of the function is a line.



Additional Example 2:

Find the intercepts of the graph of the function $f(x) = -\frac{2}{3}x - \frac{11}{3}$.

Solution:

To find the x-intercepts, solve the equation $f(x) = 0$ for x.

$$f(x) = 0$$

$$-\frac{2}{3}x - \frac{11}{3} = 0$$

$$-\frac{2}{3}x - \frac{11}{3} + \frac{11}{3} = 0 + \frac{11}{3}$$

$$-\frac{2}{3}x = \frac{11}{3}$$

$$-\frac{\not{3}}{\not{2}} \cdot \frac{\not{2}}{\not{3}}x = -\frac{\not{3}}{2} \cdot \frac{11}{\not{3}}$$

$$x = -\frac{11}{2}$$

To find the y-intercept, find $f(0)$.

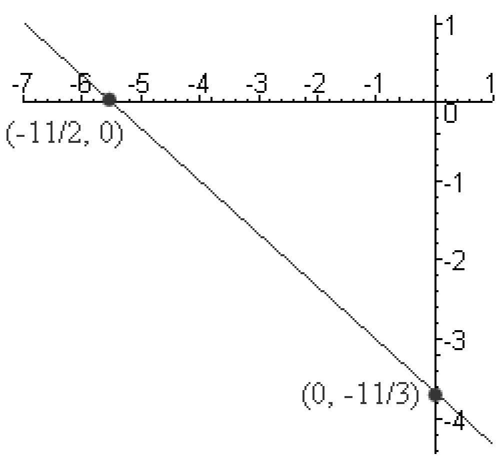
$$f(x) = -\frac{2}{3}x - \frac{11}{3}$$

$$f(0) = -\frac{2}{3} \cdot 0 - \frac{11}{3}$$

$$= 0 - \frac{11}{3}$$

$$= -\frac{11}{3}$$

The x-intercept is $-\frac{11}{2}$ and the y-intercept is $-\frac{11}{3}$. The graph of the function a line.



Additional Example 3:

Find the intercepts of the graph of the function $f(x) = x^3 - 3x^2 + 2x$.

Solution:

To find the x-intercepts, solve the equation $f(x) = 0$ for x.

$$f(x) = 0$$
$$x^3 - 3x^2 + 2x = 0$$
$$x\left(x^2 - 3x + 2\right) = 0$$
$$x(x-1)(x-2) = 0$$

$x = 0$ or $x - 1 = 0$ or $x - 2 = 0$

$x - 1 + 1 = 0 + 1$ $x - 2 + 2 = 0 + 2$

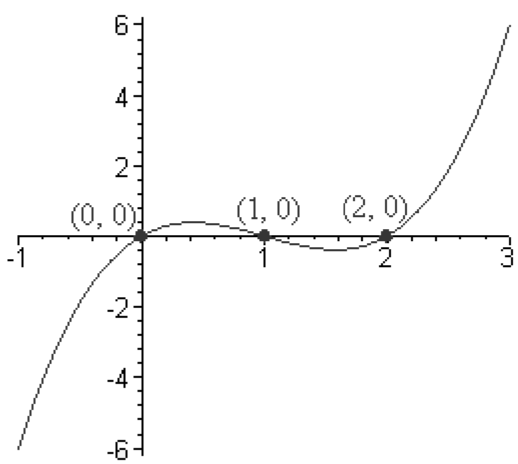
$x = 1$ $x = 2$

To find the y-intercept, find $f(0)$.

$$f(x) = x^3 - 3x^2 + 2x$$
$$f(0) = 0^3 - 3(0)^2 + 2(0) = 0$$

The x-intercepts are 0, 1, and 2. The y-intercept is 0.

The graph is shown below.



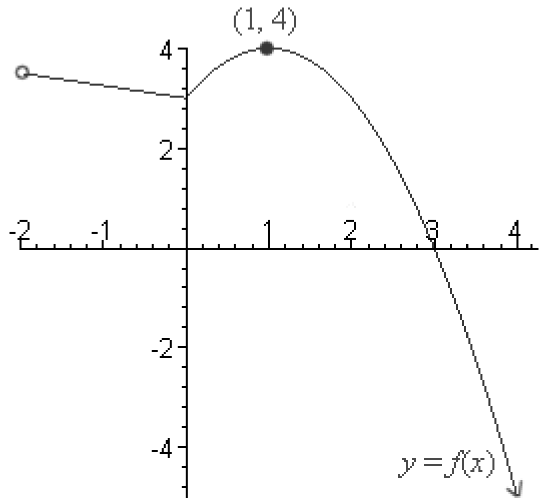
Additional Example 4:

For the function f whose graph is shown above, answer the following questions.

(a) On what interval(s) is f increasing? (b) On what interval(s) is f decreasing?

(c) Does the function have a maximum value? If so, what is the maximum value?

(d) Does the function have a minimum value? If so, what is the minimum value?



Solution:

(a) The function f is increasing on an interval where the graph is rising. Inspect the graph to find intervals of increase.

The function is increasing on the interval $(0,1)$.

(b) The function f is decreasing on an interval where the graph is falling. Inspect the graph to find intervals of decrease.

The function is decreasing on the intervals $(-2,0)$ and $(1,\infty)$.

(c) The function f has a maximum value if the graph has a highest point. Inspect the graph to determine if the function has a maximum value.

The point $(1,4)$ is the highest point on the graph of f. Thus, the function has a maximum value. The maximum value of the function is $4 = f(1)$.

(d) The function f has a minimum value if the graph has a lowest point. Inspect the graph to determine if the function has a minimum value.

The graph of f does not have a lowest point. Thus, the function does not have a minimum value.

Additional Example 5:

Determine whether the function $f(x) = 3x^4 - 2x^2 + 5$ is even, odd, or neither.

Solution:

Substitute $-x$ for x.

$$\begin{aligned} f(x) &= 3x^4 - 2x^2 + 5 \\ f(-x) &= 3(-x)^4 - 2(-x)^2 + 5 \\ &= 3x^4 - 2x^2 + 5 \\ &= f(x). \end{aligned}$$

If for each x in the domain of f, $f(-x) = f(x)$, then f is an even function. Thus, the given function is even.

Additional Example 6:

Determine whether the function $f(x) = 10x^5 - 2x$ is even, odd, or neither.

Solution:

Substitute $-x$ for x.

$$\begin{aligned} f(x) &= 10x^5 - 2x \\ f(-x) &= 10(-x)^5 - 2(-x) \\ &= 10\left(-x^5\right) + 2x \\ &= -10x^5 + 2x \\ &= -\left(10x^5 - 2x\right) \\ &= -f(x) \end{aligned}$$

If for each x in the domain of f, $f(-x) = -f(x)$, then f is an odd function. Thus, the given function is odd.

Additional Example 7:

Given that the function f is odd and $(2, -5)$ is a point on the graph of f, find another point on the graph.

Solution:

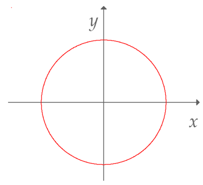
Since we are given that $(2,-5)$ is a point on the graph of f, 2 is in the domain of f and $f(2)=-5$.

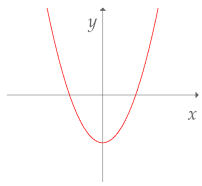
For an odd function f, for each x in the domain of f, $-x$ is also in the domain and $f(-x)=-f(x)$. Since 2 is in the domain of f, so is -2 and $f(-2)=-f(2)=-(-5)=5$.

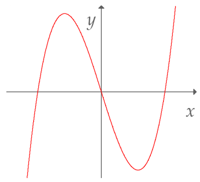
Another point on the graph of f is the point $(-2,5)$.

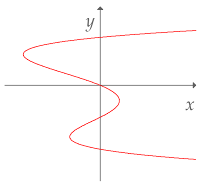
Exercise Set 1.2: Functions and Graphs

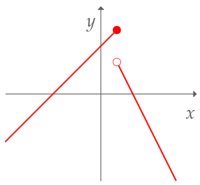
Determine whether or not each of the following graphs represents a function.

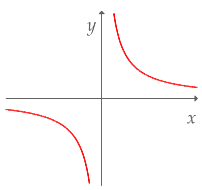
1. 

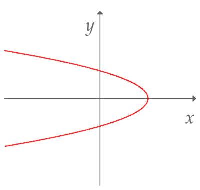
2. 

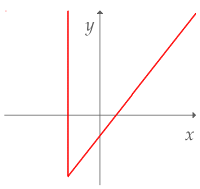
3. 

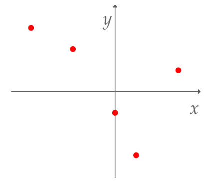
4. 

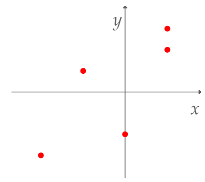
5. 

6. 

7. 

8. 

9. 

10. 

For each set of points,

(a) Graph the set of points.

(b) Determine whether or not the set of points represents a function. Justify your answer.

11. $\{(1, 5),\ (2, 4),\ (-3, 4),\ (2, -1),\ (3, 6)\}$

12. $\{(-3, 2),\ (1, 2),\ (0, -3),\ (2, 1),\ (-2, 1)\}$

Answer the following.

13. Analyze the coordinates in each of the sets above. Describe a method of determining whether or not the set of points represents a function <u>without</u> graphing the points.

14. Determine whether or not each set of points represents a function <u>without</u> graphing the points. Justify each answer.

(a) $\{(-7, 3),\ (3, -7),\ (1, 5),\ (5, 1),\ (-2, 1)\}$

(b) $\{(6, 3),\ (-4, 3),\ (2, 3),\ (-3, 3),\ (5, 3)\}$

(c) $\{(3, 6),\ (3, -4),\ (3, 2),\ (3, -3),\ (3, 5)\}$

(d) $\{(-2, -5),\ (-5, 2),\ (2, 5),\ (5, -2),\ (5, 2)\}$

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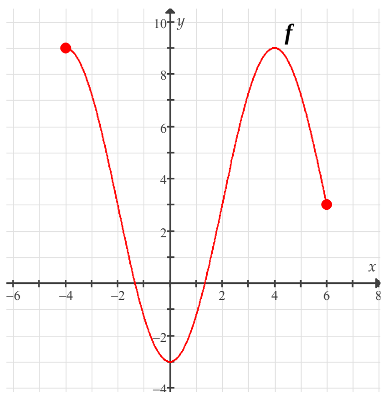
Exercise Set 1.2: Functions and Graphs

Answer the following.

15. The graph of $y = f(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) Find the *y*-intercept(s) of the function.

(d) Find the following function values:
$f(-2)$; $f(0)$; $f(4)$; $f(6)$

(e) For what value(s) of *x* is $f(x) = 9$?

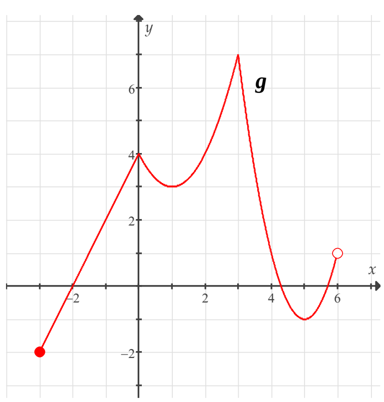
(f) On what interval(s) is *f* increasing?

(g) On what interval(s) is *f* decreasing?

(h) What is the maximum value of the function?

(i) What is the minimum value of the function?

16. The graph of $y = g(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) Find the *y*-intercept(s) of the function.

(d) Find the following function values:
$g(-2)$; $g(0)$; $g(1)$; $g(3)$; $g(6)$

(e) For what value(s) of *x* is $g(x) = -2$?

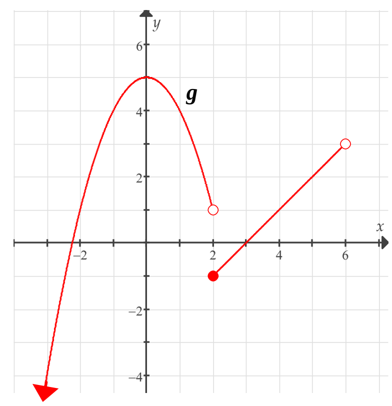
(f) On what interval(s) is *g* increasing?

(g) On what interval(s) is *g* decreasing?

(h) What is the maximum value of the function?

(i) What is the minimum value of the function?

17. The graph of $y = g(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) How many *x*-intercept(s) does the function have?

(d) Find the following function values:
$g(-2)$; $g(0)$; $g(2)$; $g(4)$; $g(6)$

(e) Which is greater, $g(-2)$ or $g(3)$?

(f) On what interval(s) is *g* increasing?

(g) On what interval(s) is *g* decreasing?

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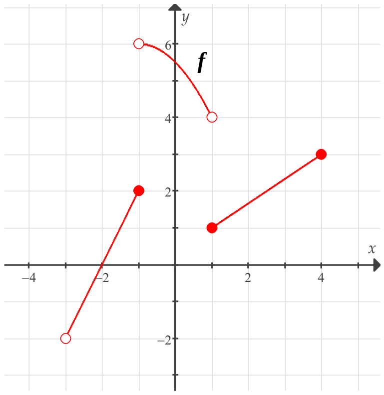
Exercise Set 1.2: Functions and Graphs

18. The graph of $y = f(x)$ is shown below.



(a) Find the domain of the function. Write your answer in interval notation.

(b) Find the range of the function. Write your answer in interval notation.

(c) Find the *x*-intercept(s) of the function.

(d) Find the following function values:
$f(-3)$; $f(-2)$; $f(-1)$; $f(1)$; $f(4)$

(e) Which is smaller, $f(0)$ or $f(3)$?

(f) On what interval(s) is *f* increasing?

(g) On what interval(s) is *f* decreasing?

For each of the following functions:

(c) **State the domain of the function. Write your answer in interval notation.**

(d) **Find the intercepts of the function.**

(e) **Choose *x*-values corresponding to the domain of the function, calculate the corresponding *y*-values, plot the points, and draw the graph of the function.**

19. $f(x) = -\frac{3}{2}x + 6$

20. $f(x) = \frac{2}{3}x - 4$

21. $h(x) = 3x - 5, \ -1 \le x < 3$

22. $h(x) = -2x, \ -3 < x \le 2$

23. $g(x) = |x - 3|$

24. $g(x) = |x| - 4$

25. $f(x) = \sqrt{x - 3}$

26. $f(x) = \sqrt{5 - x}$

27. $F(x) = x^2 - 4x$

28. $G(x) = (x - 3)^2 + 1$

29. $f(x) = x^3 + 1$

30. $f(x) = x^4 - 16$

31. $g(x) = \frac{12}{x}$

32. $h(x) = -\frac{8}{x}$

For each of the following piecewise-defined functions:

(a) **State the domain of the function. Write your answer in interval notation.**

(b) **Find the *y*-intercept of the function.**

(c) **Choose *x*-values corresponding to the domain of the function, calculate the corresponding *y*-values, plot the points, and draw the graph of the function.**

33. $f(x) = \begin{cases} 2x + 4, & \text{if } -2 \le x < 1 \\ -x + 3, & \text{if } 1 \le x \le 5 \end{cases}$

34. $f(x) = \begin{cases} \frac{1}{3}x + 2, & \text{if } -3 \le x \le 0 \\ -4x + 3, & \text{if } x > 0 \end{cases}$

35. $f(x) = \begin{cases} 3, & \text{if } x < -2 \\ -5, & \text{if } x \ge -2 \end{cases}$

36. $f(x) = \begin{cases} -4, & \text{if } -5 \le x < 1 \\ 2, & \text{if } 1 \le x \le 3 \end{cases}$

37. $f(x) = \begin{cases} 4, & \text{if } x < 0 \\ x^2 + 1, & \text{if } x \ge 0 \end{cases}$

38. $f(x) = \begin{cases} 3 - x^2, & \text{if } x \le 1 \\ -3, & \text{if } x > 1 \end{cases}$

39. $f(x)=\begin{cases} x, & \text{if } x \le -3 \\ x^2, & \text{if } -3 < x < 2 \\ 4, & \text{if } x \ge 2 \end{cases}$

40. $f(x)=\begin{cases} x^2-5, & \text{if } x < 0 \\ 1, & \text{if } 0 \le x \le 3 \\ 2-x, & \text{if } x > 3 \end{cases}$

Answer the following.

41. **(a)** If a function is odd, then it is symmetric with respect to the __________. (x-axis, y-axis, or origin?)

(b) If a function is even, then it is symmetric with respect to the __________. (x-axis, y-axis, or origin?)

42. **(a)** If a function is symmetric with respect to the y-axis, then the function is __________. (Odd, even, both, or neither?)

(b) If a function is symmetric with respect to the origin, then the function is __________. (Odd, even, both or neither?)

43. Suppose that $y = f(x)$ is an odd function and that $(-3, 6)$ is a point on the graph of *f*. Find another point on the graph.

44. Suppose that $y = f(x)$ is an even function and that $(2, -7)$ is a point on the graph of *f*. Find another point on the graph.

Determine whether each of the following functions is even, odd, both or neither.

45. $f(x) = x^3 - 5x$

46. $f(x) = x^2 + 3x$

47. $f(x) = x^4 + 2x^2$

48. $f(x) = x^5 + 2x^3$

49. $f(x) = 2x^3 + x^2 - 5x + 1$

50. $f(x) = 3x^6 + \dfrac{2}{x^2}$