Math 1314 Marginal Functions in Economics

## **Marginal Cost**

Suppose a business owner is operating a plant that manufactures a certain product at a known level. Sometimes the business owner will want to know how much it costs to produce one more unit of this product. The cost of producing this additional item is called the **marginal cost**.

**Example 1**: Suppose the total cost in dollars per week by ABC Corporation for producing its best-selling product is given by  $C(x) = 12,000 + 2000x - 0.3x^2$ . Find the actual cost of producing the  $101^{\text{st}}$  item.

The cost of producing the $(x + 1)^{\text{st}}$  item can be found by computing the average rate of change, that is by computing  $\frac{C(101) - C(100)}{101 - 100}$ .

Note that 
$$\frac{C(101) - C(100)}{101 - 100} = \frac{C(x+h) - C(x)}{(x+h) - x}$$
 where  $x = 100$  and  $h = 1$ .

The right hand side of this equation is the formula for average rate of change of the cost function. This will give us the actual cost of producing the next item. However, it is often inconvenient to use. For this reason, marginal cost is usually approximated by the instantaneous rate of change of the total cost function evaluated at the specific point of interest. That is to say, we'll find the derivative and substitute in our point of interest.

**Example 2**: Suppose the total cost in dollars per week by ABC Corporation for producing its best-selling product is given by  $C(x) = 12,000 + 2000x - 0.3x^2$ . Find C'(100).

Note that the answers for examples 1 and 2 are very close. This shows you why we can work with the derivative of the cost function rather than the average rate of change. The derivative will be much easier for us to work with. So, we'll define the **marginal cost function** as the derivative of the total cost function.

You will find that by a marginal function, we mean the derivative of the function. So, the marginal cost function is the derivative of the cost function, the marginal revenue function is the derivative of the revenue function, etc.

**Example 3**: A company produces noise-canceling headphones. Management of the company has determined that the total daily cost of producing *x* headsets can be modeled by the function  $C(x) = 0.0001x^3 - 0.04x^2 + 150x + 25,000$ . Find the marginal cost function. Use the marginal cost function to approximate the actual cost of producing the  $21^{\text{st}}$ ,  $101^{\text{st}}$  and  $181^{\text{st}}$  headsets.

## Average Cost and Marginal Average Cost

Suppose C(x) is the total cost function for producing *x* units of a certain product. If we divide this function by the number of units produced, *x*, we get the **average cost function**. We denote this function by  $\overline{C}(x)$ . Then we can express the average cost function as  $\overline{C}(x) = \frac{C(x)}{x}$ . The derivative of the average cost function is called the **marginal average cost**.

**Example 4**: A company produces office furniture. Its management estimates that the total annual cost for producing *x* of its top selling executive desks is given by the function C(x) = 600x + 120,000. Find the average cost function. What is the average cost of producing 3000 desks? What happens to  $\overline{C}(x)$  when *x* is very large?

## **Marginal Revenue**

We are often interested in revenue functions, as well. The basic formula for a revenue function is given by R(x) = px where x is the number of units sold and p is the price per unit. Often p is given in terms of a demand function in terms of x, when we can then substitute into R(x). The derivative of R(x) is called the **marginal revenue function**.

**Example 5**: A company estimates that the demand for its product can be expressed as p = -.05x + 600, where *p* denotes the unit price and *x* denotes the quantity demanded. Find the revenue function. Then find the marginal revenue function. Use the marginal revenue function to approximate the actual revenue realized on the sale of the 4001<sup>st</sup> item.

## **Marginal Profit**

The final function of interest is the profit function. This function can be expressed as P(x) = R(x) - C(x), where R(x) is the revenue function and C(x) is the cost function. As before, we will find the marginal function by taking the derivative of the function, so the **marginal profit function** is the derivative of P(x). This will give us a good approximation of the profit realized on the sale of the  $(x + 1)^{st}$  unit of the product.

**Example 6**: A company estimates that the cost to produce *x* of its products is given by the function  $C(x) = 0.000003x^3 - 0.08x^2 + 500x + 250,000$  and the demand function is given by p = 600 - 0.8x. Find the profit function. Then find the marginal profit function. Use the marginal profit function to compute the actual profit realized on the sale of the 51<sup>st</sup> unit.

**Example 7**: The weekly demand for a certain brand of DVD player is given by  $p = -.02x + 300, 0 \le x \le 15,000$ , where *p* gives the wholesale unit price in dollars and *x* denotes the quantity demanded. The weekly cost function associated with producing the DVD players is given by  $C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70,000$ . Compute C'(3000), R'(3000) and P'(3000). Interpret your results.

From this lesson, you should be able to

Explain what a marginal cost (revenue, profit) function is, and what it is used for Find a marginal cost function and use it to approximate the cost of producing the

 $x + 1^{st}$  item

Find an average cost function

Find  $\lim \overline{C}(x)$  and explain what it means

Find a revenue function.

Find a marginal revenue function and use it to approximate the revenue realized on the sale of the  $x + 1^{st}$  item

Find a profit function

Find a marginal profit function and use it to approximate the profit realized on the sale of the  $x + 1^{st}$  item