Math 1314
Some Applications of the Derivative

## Basic Applications

## Equations of Tangent Lines

The first applications of the derivative involve finding the slope of the tangent line and writing equations of tangent lines.

Example 1: Find the slope of the line tangent to $f(x)=5 x+3 x^{2}$ at the point $(-1,-2)$.

Example 2: Find the equation of the line tangent to $f(x)=\sqrt{x^{2}+4 x-8}$ at the point (2, 2).

Example 3: Find the equation of the line tangent to $f(x)=x^{3} e^{5 x}$ at the point $\left(1, e^{5}\right)$.

## Horizontal Tangent Lines, etc.

Some other basic applications involve finding where the slope of the tangent line is equal to a given number.

Example 4: Find all values of $x$ for which the line tangent to $f(x)=2 x^{3}-9 x^{2}-24 x+15$ is horizontal.

Example 5: Find all values of $x$ for which the slope of the line tangent to $f(x)=5 x+4 \ln x$ is horizontal.

Example 6: Find all values of $x$ for which the slope of the line tangent to $f(x)=x^{3}-5 x^{2}+4 x+8$ is -6 .

## Rates of Change

Sometimes we're interested in finding the rate of change of a function at a specific point.
Example 7: The Chamber of Commerce commissioned a study on population growth, which indicated that the city's population will likely grow according to the function $P(t)=125,000+80 t^{\frac{3}{2}}+550 t$, where $P$ represents the population $t$ years from now. How fast will the population be increasing in 4 years? How fast will it be increasing in 9 years?

Example 8: The membership at a fitness center can be approximated by the function $N(t)=106(75+5 t)^{\frac{2}{3}}$, where $N$ is the number of members at the beginning of week $t$. How fast was the club's membership increasing at the beginning of the $27^{\text {th }}$ week? What was the membership at the beginning of the $27^{\text {th }}$ week?

Example 9: A species of fish faces extinction, and conservation measures have been employed to ensure the survival of the species. The conservation group believes that the population will grow according to the function $N(t)=2 t^{3}-3 t^{2}+6 t+1200$, where $N$ denotes the number of fish in the population and $t$ represents the number of years after the measure were implemented. Compute $N^{\prime \prime}(8)$ and explain what it means.

## Velocity and Acceleration

A common use of rate of change is to describe the motion of an object. The function gives the position of the object with respect to time, so it is usually a function of $t$ instead of $x$. If the object changes position over time, we can compute its rate of change, which we refer to as velocity. We can find either the average rate of change or the instantaneous rate of change, depending on the question posed. The average rate of change will be the difference quotient, $\frac{f(x+h)-f(x)}{h}$. The instantaneous rate of change will be the derivative of the position function.

Velocity can be positive, negative or zero. If you throw a rock up in the air, its velocity will be positive while it is moving upward and will be negative while it is moving downward. We refer to the absolute value of velocity as speed.

When you accelerate while driving, you are increasing your speed. This means that you are changing your rate of change. Acceleration, then, is the derivative of velocity - the rate of change of your rate of change. It follows that the second derivative of a position functions gives an acceleration function.

So, if position is given by $f(t)$, and instantaneous velocity is given by $v(t)=f^{\prime}(t)$, acceleration is given by $a(t)=f^{\prime \prime}(t)$.

Example 10: A ball is dropped from a height of 80 feet. Its height $s$ (in feet) at time $t$ (in seconds) is given by $s(t)=-16 t^{2}+80$. Find the average velocity over the interval [1, 3].

Example 11: At $t=0$, a diver jumps from a diving board that is 32 feet above the water. Her position is given by $s(t)=-16 t^{2}+16 t+32$, where $t$ is measured in seconds and $s$ is measured in feet. When does she hit the water? What is her velocity when she hits the water?

From this lesson you should be able to
Find the slope of a tangent line at a point
Write an equation of a tangent line at a point
Determine values of $x$ for which the first derivative is zero (i.e., the tangent line is horizontal)
Determine values of $x$ for which the first derivative is some constant, $k$ Solve problems involving velocity and acceleration
Solve problems involving other rates of change

