Math 1314
The Chain Rule

In this lesson, you will learn the last of the basic rules for finding derivatives, the chain rules.

Example 1: Decompose $h(x) = \left(4x^3 + 2x - 1\right)^5$ into functions $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$.

Rule 9: The Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Example 2: Find the derivative if $f(x) = \left(2x^4 + 7\right)^5$.

Example 3: Find the derivative if $f(x) = \sqrt{5x^2 + 1}$.

Example 4: Find the derivative if $f(x) = \dfrac{-9}{(3x^2 - 4)^5}$.

We can also apply the chain rule in problems involving the exponential function and the logarithmic function.

Rule 10: The Chain Rule (Exponential Function)

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

Example 5: Find the derivative: $f(x) = 5e^{2x}$.

Rule 11: The Chain Rule (Logarithmic Function)

Rule 10: $\frac{d}{dx}\left[\ln|f(x)|\right] = \frac{f'(x)}{f(x)}$, provided $f(x) > 0$

Example 6: Find the derivative: $f(x) = \ln(4x^2 + 1)$.

Sometimes it is helpful to use the properties of logarithms to simplify a problem before we find the derivative:

Example 7: Find the derivative: $f(x) = \ln\left(3x^2\right)$

Example 8: Find the derivative: $f(x) = \ln\left[\left(2x^2 + 1\right)^4 \left(5x^2 - 1\right)^3\right]$

We can also use the chain rules together with either the product rule or the quotient rule.

Example 9: Find the derivative: $f(x) = x^3 e^{2x}$

Example 10: Find the derivative: $f(x) = x^5 \ln\left(x^2 - 7\right)$

Example 11: Find the derivative of $f(x) = x^2\left(3x^2 - 5\right)^3$.

From this lesson, you should be able to

- Apply the chain rules to appropriate problems to find derivatives
- Use the chain rule, together with other rules, to find derivatives
- Use log properties to simplify log problems before finding derivatives