Math 1314 The Derivative

The Limit Definition of the Derivative

We now address the first of the two questions of calculus, the tangent line question.

We are interested in finding the slope of the tangent line at a specific point.



We could attempt to answer this question by graphing the function and its tangent line at the point of interest. However, with many functions, we'd get an approximation at best. We need a way to find the slope of the tangent line analytically for every problem that will be exact every time.



Consider this function:

We can draw a secant line across the curve, then take the coordinates of the two points on the curve, P and Q, and use the slope formula to approximate the slope of the tangent line.

Now suppose we move point Q closer to point P. When we do this, we'll get a better approximation of the slope of the tangent line.



Suppose we move point Q even closer to point P. We get an even better approximation. We are letting the distance between P and Q get smaller and smaller. What does this sound like?



Now let's give these two points names. We'll express them as ordered pairs.



Now we'll apply the slope formula to these two points.

This expression is called a **difference quotient**.

The last thing that we want to do is to let the distance between P and Q get arbitrarily small, so we'll take a limit.

This gives us the definition of the slope of the tangent line.

Definition: The slope of the tangent line to the graph of f at the point P(x, f(x)) is given by

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

The difference quotient gives us the **average rate of change.** We find the **instantaneous rate of change** when we take the limit of the difference quotient.

The derivative of f with respect to x is the function f'(read "f prime") defined by

 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. The domain of f'(x) is the set of all x for which the limit exists.

The Four-Step Process for Finding the Derivative

Now that we know what the derivative is, we need to be able to find the derivative of a function. We'll use an algebraic process to do so. We'll use a 4-Step process to find the derivative. The steps are as follows:

1. Find f(x+h).

2. Find
$$f(x+h) - f(x)$$
.

- 3. Form the difference quotient $\frac{f(x+h) f(x)}{h}$.
- 4. Find the limit of the difference quotient as h gets close to 0: $\lim_{h \to 0} \frac{f(x+h) f(x)}{h}$

So,
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
.

Example 1: Find the rule for the slope of the tangent line for the function f(x) = 5x - 3.

Step 1:

Step 2:

Step 3:

Step 4:

Example 2: Use the four-step process to find the derivative of $f(x) = 3x^2$.

Step 1:

Step 2:

Step 3:

Step 4:

Example 3: Find the derivative of $f(x) = \frac{1}{5x}$.

Example 4: Find the derivative if $f(x) = -2x^2 + 6x + 3$.

Example 5: Suppose $f(x) = 3x^2 - 4x - 1$. Find the average rate of change of *f* over the interval [2, 5].

From this lesson, you should be able to Explain what a derivative is State the limit definition of the derivative Use the four-step process to find the derivative of a polynomial function Find the average rate of change