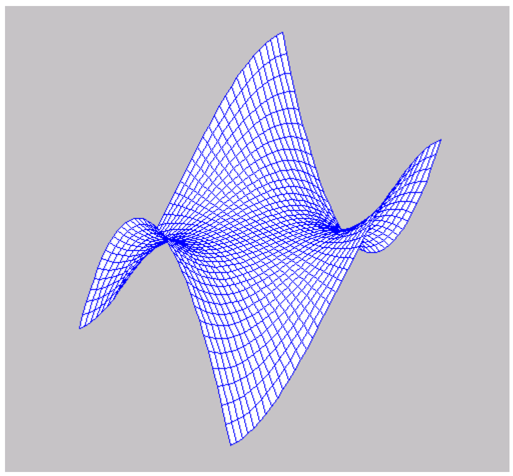
Math 1314
Maxima and Minima of Functions of Several Variables

Relative Extrema of Functions of Two Variables

We learned to find the maxima and minima of a function of a single variable earlier in the course. Although we did not use it much, we had a second derivative test to determine whether a critical point of a function was a maximum or a minimum, or possibly that the test was not conclusive. We will use a similar technique to find relative extrema of a function of several variables.

Since the graphs of these functions are more complicated, determining relative extrema is also more complicated. At a specific critical number, we can have a max, a min, or something else. That "something else" is called a saddle point.



The method for finding relative extrema is very similar to what you did earlier in the course.

First, find the first order partial derivatives and set them equal to zero. You will have a system of equations in two variables which you will need to solve to find the critical points.

Second, you will apply the second derivative test. To do this, you must find the second order partial derivatives. Let $D(x, y) = f_{xx} f_{yy} - (f_{xy})^2$. You will compute $D(a,b)$ for each critical point (a,b). Then you can apply **the second derivative test for functions of two variables**:

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a <u>relative maximum</u> at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a <u>relative minimum</u> at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

Example 1: Find the relative extrema of the function $f(x, y) = x^2 + y^2$.

Example 2: Find the relative extrema of the function $f(x, y) = x^3 + y^2 - 6x^2 - 8y - 15x + 7$.

Example 3: Find the relative extrema of the function $f(x, y) = x^3 + 2y^2 - 2xy - 3x - 2y + 5$.

Example 4: Find the relative extrema of the function $f(x,y) = -3x^3 + 4xy - 2y^2 + 7$.

Example 5: The total daily revenue (in dollars) that a publishing company realizes in publishing and selling its English language dictionary is given by $R(x, y) = -.005x^2 - .003y^2 - .002xy + 20x + 17y$ where x denotes the number of deluxe copies and y denotes the number of standard copies published and sold. The total daily cost of publishing these dictionaries is given by $C(x, y) = 6x + 3y + 200$ dollars. Determine the number of standard copies and the number of deluxe copies that the publishing company should publish per day to maximize its profits. What is the maximum profit realizable?

From this section, you should be able to

Find relative extrema of functions of two variables

Solve word problems involving extrema of functions of two variables