Math 1314
Partial Derivatives

Partial Derivatives

When we are asked to find the derivative of a function of a single variable, $f(x)$, we know exactly what to do. However, when we have a function of two variables, there is some ambiguity. We can find the slope of the tangent line at a point *P* from an infinite number of directions. We will only consider two directions, either parallel to the *x* axis or parallel to the *y* axis. When we do this, we fix one of the variables. Then we can find the derivative with respect to the other variable.

So, if we fix *y*, we can find the derivative of the function with respect to the variable *x*. And if we fix *x*, we can find the derivative of the function with respect to the variable *y*.

These derivatives are called **partial derivatives**.

First Order Partial Derivatives

We will use two different notations:

Example 1: Find the first order partial derivatives of the function $f(x, y) = x^2 - 4xy^2 + 3y^2$.

Example 2: Find the first order partial derivatives of the function $f(x,y)=\dfrac{2xy}{x^2+3y^2}$.

Example 3: Find the first order partial derivatives of the function $f(x,y)=\left(x^2+xy-5y^2\right)^3$.

Example 4: Find the first order partial derivatives of the function $f(x,y) = e^{x^2+2y^2}$.

Example 5: Find the first order partial derivatives of the function $f(x,y) = \ln\left(5x^2 + 2y^2\right)$

Second Order Partial Derivatives

Sometimes we will need to find the second order partial derivatives. To find a second order partial derivative, you will take respective partial derivatives of the first order partial derivative. There are a total of 4 second order partial derivatives.

There are two notations, but we will only use one of them.

Example 6: Find the second order partial derivatives of the function $f(x,y) = 5x^2y^2 - 2x^2 + 15y$.

Example 7: Find the second order partial derivatives of the function
$f(x,y) = 4x^2 + 2x^3y^3 - xy + 3y^3$.

We can also evaluate a partial derivative at a given point.

Example 8: Evaluate the first order partial derivative of $f(x, y) = 2xy^3 + e^{xy}$ at the point (1, 2).

From this section, you should be able to

- Find first order partial derivatives
- Find second order partial derivatives
- Evaluate partial derivatives at a given point