Math 1314
Functions of Several Variables

So far, we have looked at functions of a single variable. In this section, we will consider functions of more than one variable. You are already familiar with some examples of these.

$P(x, y) = 2x + 2y$
$A(P, i, t) = P(1 + i)^t$

These formulas are functions of several variables. We have just never called them that before. We will, for the most part, limit our discussion to functions of two variables.

Functions of Two Variables

Definition: A real valued function of two variables, *f*, consists of a set *A* of ordered pairs of real numbers (*x, y*) called the domain of the function, and a rule that associates with each ordered pair in the domain of *f* one and only one real number, denoted by $z = f(x, y)$.

You will need to learn several skills using functions of several variables:

1. **Evaluating** a function of several variables at a given point.

Example 1: Suppose $f(x, y) = 5x^2y + 2xy - 6$. Compute $f(0,0)$, $f(-2,1)$ and $f(-2,-3)$.

Example 2: The volume of a cylindrical tank with radius *r* and height *h* is given by the formula $V = f(r, h) = \pi r^2 h$. Find the volume of a tank with radius 8 feet and height 14 feet.

2. **Find the domain** of a function of several variables. We'll have several kinds of situations that are similar to finding domain of functions of a single variable. Problems arise when we have rational functions, functions involving radicals and logarithmic functions.

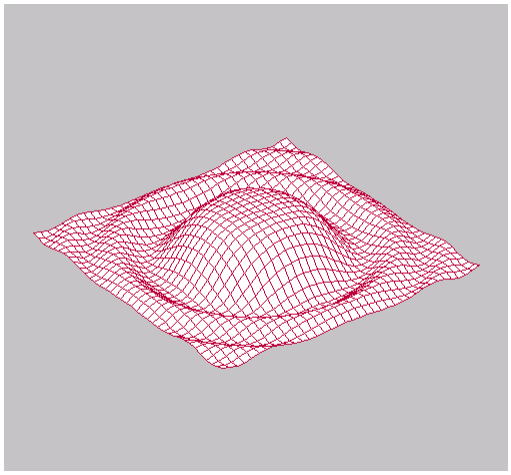
Example 3: Find the domain of the function $f(x,y) = 5x^2 - 2y^2$.

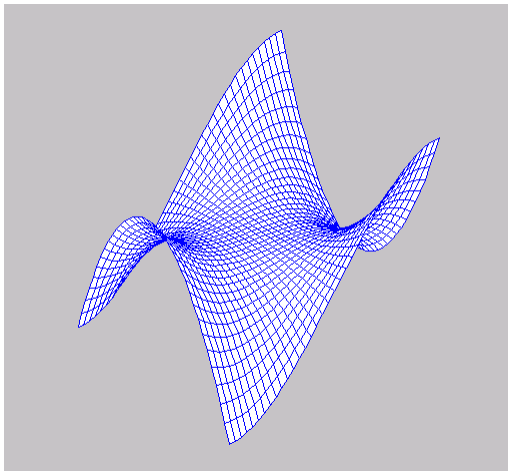
Example 4: Find the domain of the function $f(x,y) = \dfrac{6x}{3x-4y}$.

Example 5: Find the domain of the function $f(x,y) = \sqrt{25 - x^2 - y^2}$.

Example 6: Find the domain of the function $f(x,y) = \ln(5x + y)$.

Graphing functions in space is quite difficult. You will not need to do this. Here are a couple of examples of graphs of functions of two variables.



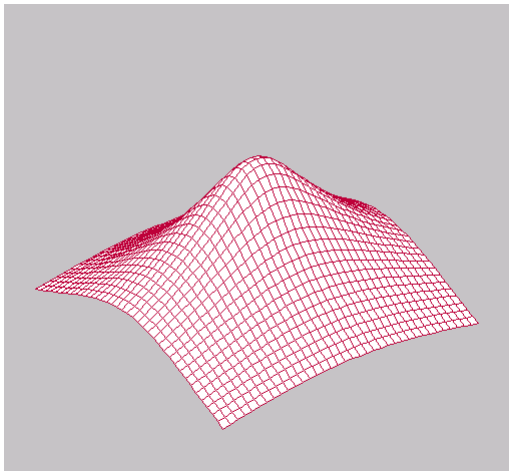


Hopefully, you'll recognize how difficult it would be to graph these manually. We usually graph functions of two variables using a computer.

3. You will need to be able to **graph level curves** of a function of two variables.

Sometimes, we need to get a snapshot of what is happening to a function. To do this, we choose a value, *c*, for z, and graph the function $f(x, y) = c$. This gives us a function in *x* and *y* only, and we are able to graph this in the coordinate plane. This is called a **trace** of the graph of *f* in the plane where $z = c$. If we then project this onto the *xy* plane, we have a level curve.

Suppose we consider this function of two variables:



The traces are horizontal "slices" at various heights. We'll graph the outlines of these in the *xy* place.

You can draw the level curves for some simple functions.

Example 7: Sketch the level curves for the function $f(x, y) = x^2 + y^2$ where c = 1, 4, 9, 16 and 25.

Example 8: Sketch the level curves for the function $f(x, y) = -x^2 + y$, where c=-2, -1, 0, 1, and 2.

From this section, you should be able to

- Evaluate a function of several variables
- Find the domain of a function of several variables
- Sketch level curves, given a function of several variables and value(s) for *c*.