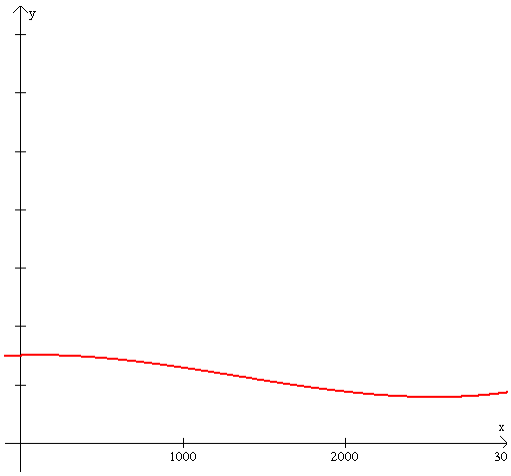
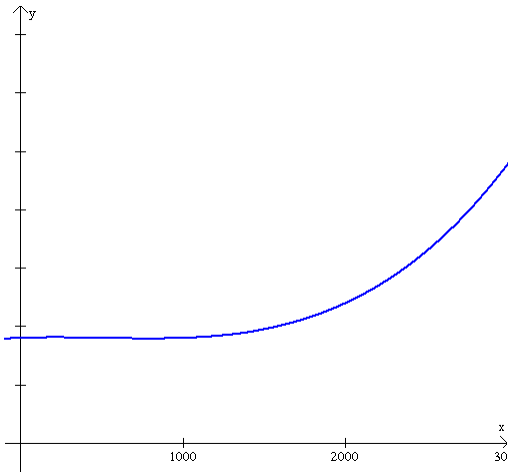
Math 1314
Area Between Two Curves

Two advertising agencies are competing for a major client. The rate of change of the client's revenues using Agency A's ad campaign is approximated by $f(x)$ below. The rate of change of the client's revenues using Agency B's ad campaign is approximated by $g(x)$ below. In both cases, x represents the amount spent on advertising. In each case, total revenue is the area under the curve.

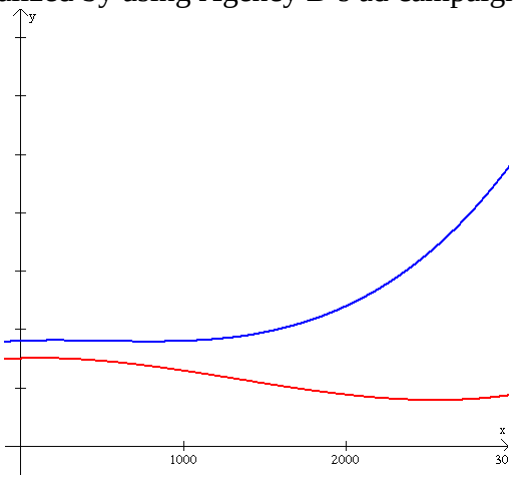
Agency A



Agency B



This graph shows the relationship between the two revenue functions. We see that one function is above the other. The area between the two functions represents the additional revenue that would be realized by using Agency B's ad campaign.

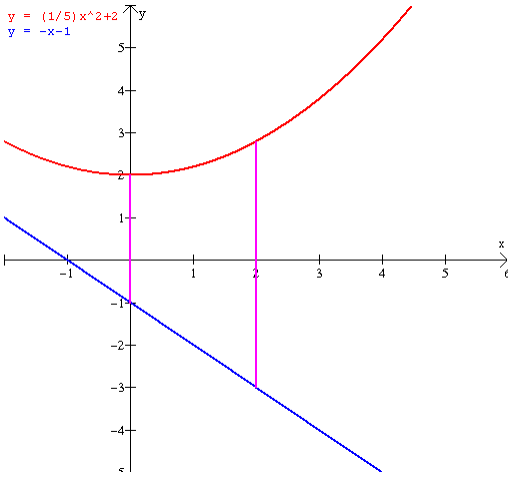


This is an example of the kinds of problems you will be able to solve with the techniques you learn in this lesson.

We can compute the area between the two curves. The general "formula" is

$$\int_a^b (\text{top function} - \text{bottom function})dx$$

Example 1: Find the area between the two curves:



Example 2: Find the area between the function $f(x) = x^3 - 4x$ and the x axis from x = -1 to x = 1.

Example 3: Find the area between the functions $f(x) = x^2 - 9x$ and $g(x) = x$.

Example 4: Find the area between the functions $f(x) = x^3$ and $g(x) = -2x$ and the vertical lines x = -1 and x = 2.

Example 5: Find the area between the functions $f(x) = e^x$ and $g(x) = \frac{1}{x}$ and the vertical lines $x = 3$ and $x = 4$.

Example 6: Find the area completely enclosed by the functions $f(x) = \sqrt{x}$ and $g(x) = x^2$.

Example 7: Without any effort to curb population growth, a government estimates that its population will grow at the rate of $45e^{.02t}$ thousand people per year. However, they believe that an education program will alter the growth rate to $-t^2 + 45$ thousand people per year over the next 5 years. How many fewer people would there be in the country if the education program is implemented and is successful?

Example 8: A consumer magazine tested two kinds of engines. One was a standard engine, and it was determined that its acceleration could be modeled by $f(t) = 6 + .7t$ feet per sec^2, t seconds after starting from rest. The acceleration of the turbo-charged model could be approximated by $g(t) = 6 + 1.6t + .05t^2$ feet per sec^2, t seconds after starting from rest. How much faster is the turbo-charged model moving than the standard model at the end of a 10 second trial?

From this lesson, you should be able to

- Find the area between two curves
  - Sketch the graphs
  - Set up the necessary integral(s)
  - Find points of intersection if necessary
  - Integrate and evaluate
- Solve word problems involving the area between two curves