Math 1314 Evaluating Definite Integrals

We will sometimes need these properties when computing definite integrals.

Properties of Definite Integrals

Suppose f and g are integrable functions. Then:

$$1. \quad \int_{a}^{a} f(x) dx = 0$$

2.
$$\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$$

3.
$$\int_{a}^{b} cf(x) dx = c \int_{a}^{b} f(x) dx$$

4.
$$\int_{a}^{b} [f(x) \pm g(x)] dx = \int_{a}^{b} f(x) dx \pm \int_{a}^{b} g(x) dx$$

5.
$$\int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx$$
 where $a < c < b$

We will need to use substitution on some problems:

Example 1: Evaluate $\int_{0}^{2} 6x(x^{2}+1)^{4} dx$

Example 2: Evaluate $\int_0^1 x e^{2x^2} dx$

Example 3: Evaluate
$$\int_{1}^{2} \frac{x^2}{5x^3+1} dx$$

Example 4: Evaluate
$$\int_{1}^{4} \frac{2x}{\sqrt{x^2+1}} dx$$

Example 5: Evaluate
$$\int_{1}^{5} \frac{(\ln x)^{5}}{x} dx$$

Applications

Example 6: The daily marginal profit function associated with the production and sales of a product is $P'(x) = -.0003x^2 + .08x + 25$ where *x* denotes the number of units manufactured and sold daily and P'(x) is measured in dollars per unit. Find the total profit realizable from the manufacture and sale of 300 units per day.

Example 7: Suppose you are driving a car and that your velocity can be approximated by $v(t) = 2t\sqrt{16-t^2}$, where *t* is measured in seconds and *v* is measured in feet per second. How far will you travel in the 4 seconds from t = 0 to t = 4?

The Average Value of a Function

We can use the definite integral to find the average value of a function.

Suppose *f* is an integrable function on the interval [*a*, *b*]. Then the average value of *f* over the interval is $\frac{1}{b-a}\int_{a}^{b} f(x)dx$.

This is what average value represents:



Example 8: Find the average value of $f(x) = 2\sqrt{x}$ over the interval [1, 9].

Example 9: Find the average value of $f(x) = \frac{1}{x-1}$ on [3, 9].

From this lesson, you should be able to

Use the FTOC to compute definite integrals, including problems that require substitution
Define average value
Use the FTOC to find average value
Solve word problems using FTOC