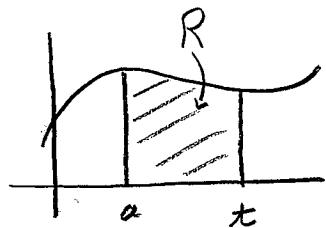
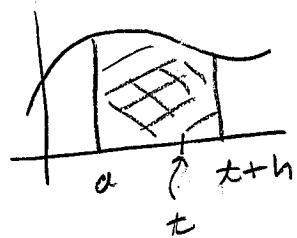


"Proof" of the Fundamental Theorem of Calculus.

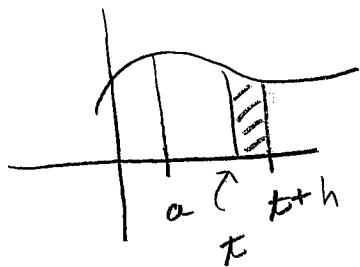
Suppose f is a non-negative function on $[a, b]$. Let $A(t)$ be the area of the region R under the graph of f from $x=a$ to $x=t$.



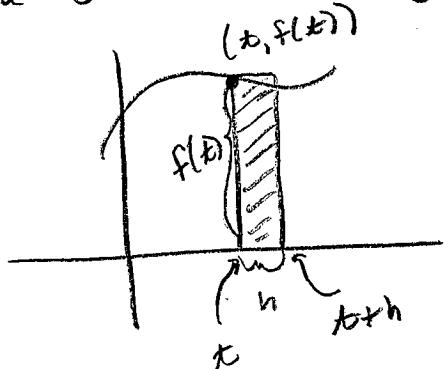
Let h be a positive number. Then $A(t+h)$ is the area under the graph of f from $x=a$ to $x=t+h$.



Then the area under the graph of f from $x=t$ to $x=t+h$ can be expressed as $A(t+h) - A(t)$.



We can approximate the area here by $n \cdot f(t)$, the area of a rectangle.



As $h \rightarrow 0$, this approximation improves. So we have $A(t+h) - A(t) \approx h \cdot f(t)$.

Dividing both sides by h , we have

$$\frac{A(t+h) - A(t)}{h} \approx \frac{h \cdot f(t)}{h}$$

$$\text{So } \frac{A(t+h) - A(t)}{h} \approx f(t)$$

Take the limit as $h \rightarrow 0$ of the left hand side.

$$\lim_{h \rightarrow 0} \frac{A(t+h) - A(t)}{h} = A'(t)$$

Note that as $h \rightarrow 0$, the approximation becomes exact, so we have

$$A'(t) = f(t).$$

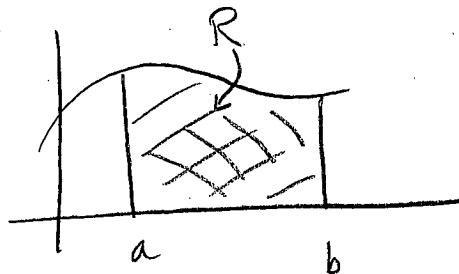
So, the area function is an antiderivative of $f(x)$.

Now, since A is an antiderivative of f ,

$$A(x) = F(x) + C$$

$$A(a) = 0, \text{ so } A(a) = F(a) + C = 0 \text{ so } C = -F(a)$$

Now consider the area of this region



The area of this region is $A(b)$ (See the definition of $A(t)$ at the beginning of the "proof.")

Then $A(b) = F(b) + C$.

Since $C = -F(a)$, we have $A(b) = F(b) - F(a)$.

The area of R is $\int_a^b f(x)dx$, so

$$A(b) = \int_a^b f(x)dx = F(b) - F(a)$$

which is what we set out to show.