Math 1314
Lesson 19
The Fundamental Theorem of Calculus

In the last lesson, we approximated the area under a curve by drawing rectangles, computing the area of each rectangle and then adding up their areas. We saw that the actual area was found as we let the number of rectangles get arbitrarily large. Computing area in this manner is very tedious, so we need another way to find the area.

The fundamental theorem of calculus allows us to do just this. It establishes a relationship between the antiderivative of a function and its definite integral.

The Fundamental Theorem of Calculus

Let *f* be a continuous function on $[a, b]$. Then $\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is any antiderivative of *f*. If you are interested in seeing why this works, see the link for the "proof" of the fundamental theorem of calculus on the class notes page.

Example 1: Suppose $f(x) = 3x + 2$. Find the area under the graph of *f* from $x = 0$ to $x = 4$.

Example 2: Evaluate $\int_0^2 (4x^2 - 3e^x)dx$

Example 3: Evaluate: $\int_1^2 (3x^2 - 6x + 5)dx$

Example 4: Evaluate: $\int_3^6 \left(\frac{4}{x} - \frac{1}{x^3} \right) dx$

Example 5: Evaluate: $\int_1^4 \frac{6x^2 - 3x + 4}{x} dx$

Example 6: Evaluate: $\int_1^{64} \sqrt{x} - \frac{1}{\sqrt[3]{x}}\, dx$

Example 7: Evaluate: $\int_0^4 (e^x - 2x + 5) dx$

Example 8: Evaluate: $\int_0^2 (x^2+2)(x-4)dx$

Example 9: Find the area of the region under the graph of $f(x) = 5x - x^2$ over the interval [0, 5].

Example 10: Find the area of the region under the graph of $f(x) = e^x - 2x$ over the interval [1, 2].

From this lesson, you should be able to

State the fundamental theorem of calculus

Use the FTOC to compute definite integrals

Use the FTOC to find area under a curve