Math 1314
Area and the Definite Integral

We are now ready to tackle the second basic question of calculus – the area question.

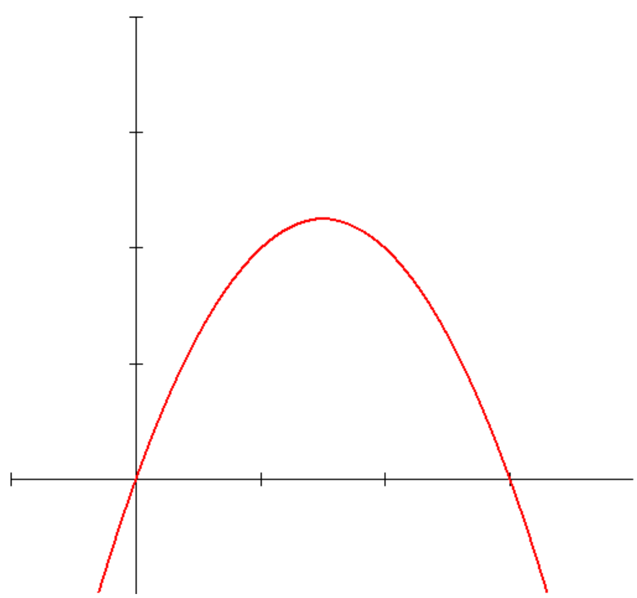
We can easily compute the area under the graph of a function so long as the shape of the region conforms to something for which we have a formula for geometry.

**Example 1**: Suppose $f(x) = 5$. Find the area under the graph of $f(x)$ from $x = 0$ to $x = 4$.

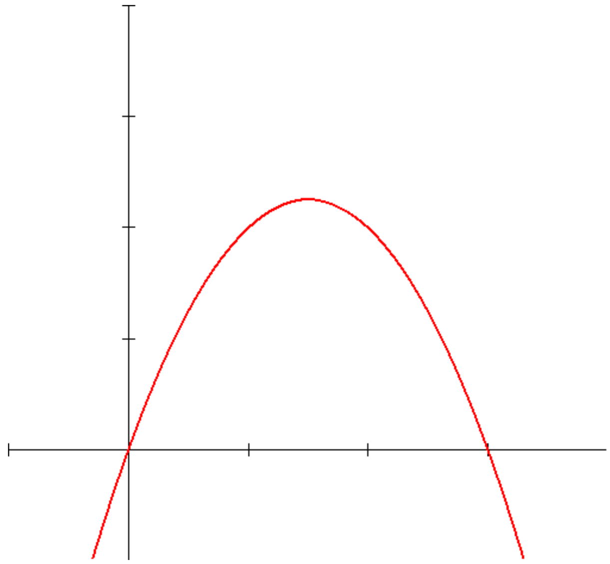
## Approximating Area Under a Curve

Now suppose the area under the curve is not something whose area can be easily computed. We'll need to develop a method for finding such an area.

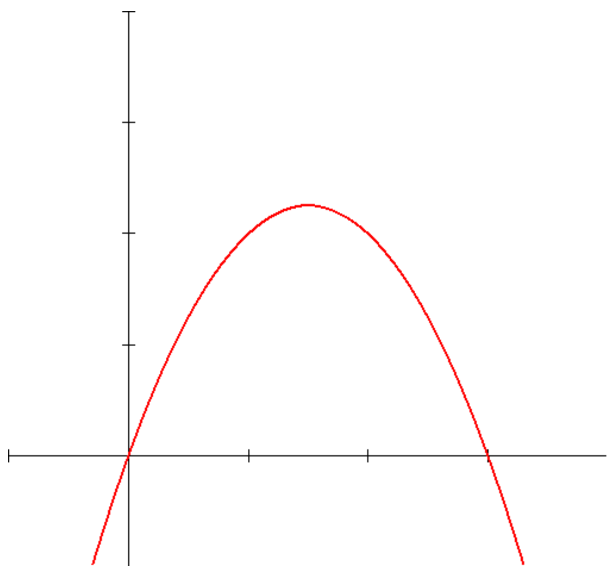
**Example 2**:



**Example 3**:



**Example 4**:



What you should see is that as the number of rectangles increases, the area we compute in this method becomes more accurate.

**The Area Under the Graph of a Function:**

Let $f$ be a nonnegative continuous function on [$a$, $b$]. Then the area of the region under the graph of $f$ is given by

$$A = \lim_{x\to\infty}[f(x_1) + f(x_2) + \ldots + f(x_n)]\Delta x$$

where $x_1, x_2, \ldots, x_n$ are arbitrary points in the interval [$a$, $b$] of equal width $\Delta x = \dfrac{b-a}{n}$.

The sums of areas of rectangles are called **Riemann sums** and are named after a German mathematician.

**Example 5**: Use left endpoints and 4 subdivisions of the interval to approximate the area under $f(x) = 3x^2 + 2$ on the interval $[0, 2]$.

**Example 6**: Use right endpoints and 4 subdivisions of the interval to approximate the area under $f(x) = 3x^2 + 2$ on the interval $[0, 2]$.

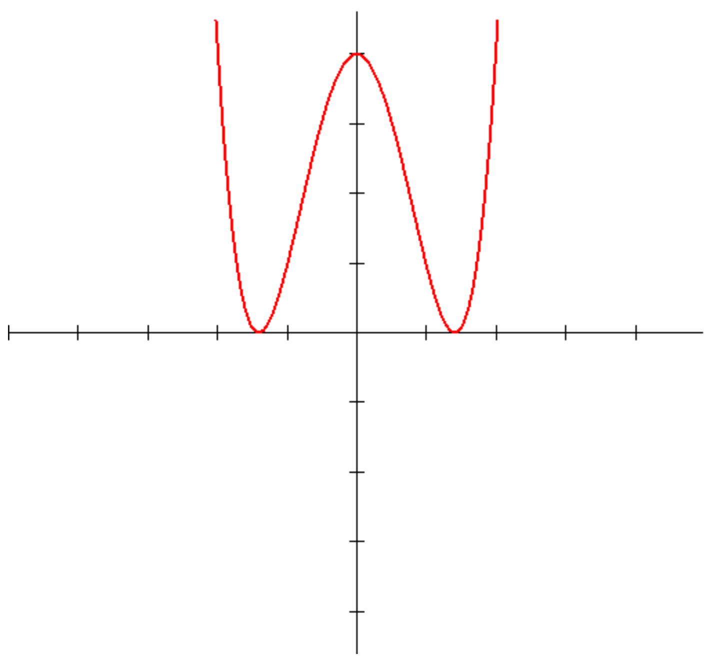
**Example 7**: Use midpoints and 4 subdivisions of the interval to approximate the area under $f(x) = 3x^2 + 2$ on the interval $[0, 2]$.

## The Definite Integral

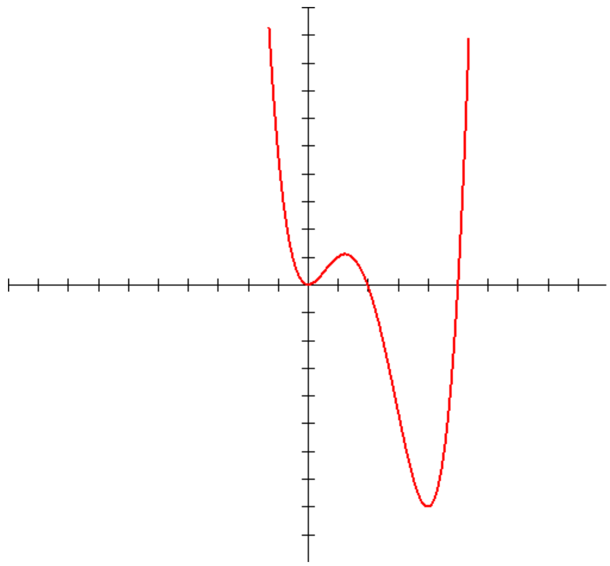
Let *f* be defined on [*a*, *b*]. If $\lim_{x\to\infty}\left([f(x_1)+f(x_2)+\ldots+f(x_n)]\Delta x\right)$ exists for all choices of representative points in the *n* subintervals of [*a*, *b*] of equal width $\Delta x=\dfrac{b-a}{n}$, then this limit is called the definite integral of *f* from *a* to *b*. The definite integral is noted by $\int_a^b f(x)dx=\lim_{x\to\infty}\left|[f(x_1)+f(x_2)+\ldots+f(x_n)]\Delta x\right|$. The number *a* is called the lower limit of integration and the number *b* is called the upper limit of integration.

A function is said to be **integrable** on [*a*, *b*] if it is continuous on the interval [*a*, *b*].

**The definite integral of a nonnegative function:**



The definite integral of a general function:



From this section, you should be able to

- Explain the procedure used to approximate area under a curve
- Use Riemann sums to approximate the area under a curve using right endpoints, left endpoints or midpoints
- Explain what we mean by definite integral of a non-negative function or a general function.