Math 1314
Lesson 14
Optimization

Now you'll work some problems where your objective is to optimize a function. That means you want to make it as large as possible or as small as possible.

The first task is to write a function that describes the situation in the problem.

Here are some suggestions to help make this easier:

1. Read the problem carefully to determine what function you are trying to find.
2. If possible, draw a picture of the situation. Choose variables for the values discussed and put them on your picture.
3. Determine if there are any formulas you need to use, such as area or volume formulas. If you have a right triangle in your picture, decide if the Pythagorean Theorem will help.

In many problems, you'll state the domain before you work the problem. Once you have the function and its domain, you'll find the critical points and see if the critical point(s) fall within the domain of the function. You can use the second derivative test to verify that you have an absolute max or an absolute min in many problems. Be sure you answer the question that is asked in each question.

Example 1: A man would like to have a rectangular shaped garden in his back yard. He has 400 feet of fencing to use to fence in the garden. Find the dimensions for the largest possible garden he can make if he uses all of the fencing.

Example 2: If you cut away equal squares from all four corners of a piece of cardboard and fold up the sides, you will make a box with no top. Suppose you start with a piece of cardboard the measures 8 feet by 10 feet. Find the dimensions of the box that will give a maximum volume.

Example 3: Suppose you want to construct a box with no top at minimum cost. The box must have a volume of 100 cubic centimeters and will have a square base. The material that will be used for the bottom of the box costs $15 per square centimeter, and the material that will be used for the sides of the box costs $10 per square centimeter. What dimensions should the box be?

Example 5: Postal regulations state that the girth plus length of a package must be no more than 104 inches if it is to be mailed through the US Postal Service. You are assigned to design a package with a square base that will contain the maximum volume that can be shipped under these requirements. What should be the dimensions of the package? (Note: girth of a package is the perimeter of its base.)

Example 7: A rancher wants to construct 3 congruent adjacent rectangular-shaped pens. He has 600 feet of fencing material to use. He will use the long side of a barn as one side of the pens, so that side will not need to be fenced. What should be the dimensions of one of the 3 pens if he uses up all of the fencing and fences in the maximum area?

If the problem asks you to optimize a rate of change, remember to use the second derivative, as in the next example:

Here, N is the number of crimes, so its derivative is the rate of change of the number of crimes. The N' is the function we wish to maximize.

Example 9: The number of major crimes committed in a city between 1997 and 2004 can be approximated by the function $N(t) = -.1t^3 + 1.5t^2 + 100,\ 0 \le t \le 7$, where N denotes the number of crimes and t denotes the year with $t = 0$ corresponding to 1997. During what year was the crime rate highest?

From this section, you should be able to

Write a function from a verbal description

Optimize a function